Symbolic Construction of Dynamic Mixed Integer Programs for Power System Management

Harry G. Kwatny, *Fellow, IEEE*, Edoe F. Mensah, Dagmar Niebur, *Member, IEEE,* Carole Teolis

***Abstract*—In this presentation, we discuss a symbolic tool, implemented in *Mathematica*©, that converts a hybrid automaton model of a power system into either a mixed logical dynamic system' (MLD) or a 'dynamic mixed integer program' (DMIP). The tool, converts any logical specification into mixed-integer formulas (IP formulas). For example the transition specification for the automaton is converted into a set of inequalities involving Boolean variables. The IP formulas can be used in decision computations using mixed integer programs or mixed integer dynamic programming. An example will be given involving controller design for a power conditioning system.**

***Index Terms*—Power System Security Assessment, Static Contingency Analysis, Symbolic Computation**

I. Introduction

The prevention of widespread power system failure remains a serious concern today. The basic physics and mathematics of power system collapse are now well known. Earlier efforts laid the groundwork for a number of investigators in the mid and late 1980's at which time a precise understanding of voltage collapse as a bifurcation of the underlying differential-algebraic equations was established, e.g. [1-4]. All of these efforts are central to understanding how power systems break down during disruption. However, the picture is not complete because the system collapse usually involves a period of discrete events associated with the action of various protection systems intended to prevent, or at least limit the scope, of any failure. It is an unfortunate fact that these systems frequently fail to achieve that goal – and worse, they sometimes amplify the effect of a small disturbance into a major outage. The Northeast blackout of August 2003 is a recent example.

The underlying issue is how do we model, analyze and synthesize systems consisting of both complex nonlinear continuous dynamics and discrete event dynamics. A power system's continuous dynamics might include a classical differential algebraic equation (DAE) model of the network with generators and loads and also continuous controllers like governors and automatic voltage regulators. Discrete event dynamics can be defined by a finite state machine that models various discrete controllers like tap-changing transformers, capacitor banks, load shedding devices and protection systems.

Thus, the system can be modeled as a hybrid automaton. While the hybrid automaton model is a convenient theoretical tool, other forms of models are far more convenient for control system design and some other computational purposes. Such models include the 'mixed logical dynamic system' (MLD) [5, 6] or a modified version that we call the 'dynamic mixed integer program' (DMIP).

In this presentation, we discuss a symbolic tool that converts a hybrid automaton model of a power system into one of these forms. The tool, implemented in *Mathematica*©, converts any logical specification into mixed-integer formulas (IP formulas). For example the transition specification for the automaton is converted into a set of inequalities involving Boolean variables. Our work extends earlier work in this area reported in [7]. Many decision problems are most naturally formulated in terms of logical specifications, but are more easily solve by mathematical programming. Consequently, the idea of reducing logical specifications into IP formulas has along history, see for example [8].

The IP formulas can be used in decision computations using mixed integer programs or mixed integer dynamic programming. Our approach derives a feedback policy based on finite, receding horizon dynamic programming. Other methods that have been proposed for hybrid systems, specifically, model predictive control, perform the computations on-line. Given the current state, they compute the optimal trajectory over a specified future time period. The computation is repeated every Δt sec. However, the feedback policy is computed once off-line and implemented in a form such as table look-up. To do this efficiently, we need to

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exploit the special structure of the power system decision problem.

In the dynamic programming approach, working backward in time, at each state it is necessary to carry out a minimization process involving continuous and integer (binary) variables to obtain the optimal control. A prototype computational method has been implemented in *Mathematica*©, which provides several tools for working with mixed variables. The problems of interest have considerable special structure that can be exploited. For example, we have many inequality constraints which implies that we should employ a constraint driven procedure. Moreover, most of the constraints are linear in binary variables. Accordingly, a specialized and novel optimization procedure was built around the function `Reduce`.

In the panel presentation, examples will be given involving controller design for a power conditioning system, a DC DC boost converter, and a power management of a small power system. In the next section we illustrate the design process for the controller design of a power conditioning system.

II. Power Conditioning system Example

The power conditioning system shown in Figure 1 is a very simple example. . This type is often used in high performance drives (weapon turrets, vehicles) to isolate a DC supply from large current demands. Its purpose is to insure that the current demand on the DC source is limited even though the load current i_L may be quite large for short periods of time.

The problem, of course, is the design of the switching strategy. A hybrid automaton model of the system without a specified control strategy is shown in Figure 2 where *q* is the capacitor charge. In this open loop configuration, the events are by an externally generated event – the switch. The proposition *s* denotes 'the switch is closed'.

The logical specification that defines the transition dynamics of the automaton is

$$\begin{aligned}&(q_1 \oplus q_2) \wedge \left(q_1(t) \wedge s \Rightarrow q_2(t^+)\right) \wedge \left(q_1(t) \wedge \neg s \Rightarrow q_1(t^+)\right)\\ &\wedge \left(q_2(t) \wedge s \Rightarrow q_1(t^+)\right) \wedge \left(q_2(t) \wedge \neg s \Rightarrow q_2(t^+)\right)\end{aligned} \tag{1}$$

Here the notation t, t^+ denotes the times just before and after the transition.

The specification (1) of the hybrid automaton with constraints is automatically converted into the set of IP inequalities using our *Mathematica*©,based tool:

$$\begin{gathered}1-\delta_{q_1^+}-\delta_{q_2^+} \ge 0\\ -1+\delta_{q_1^+}+\delta_{q_2^+} \ge 0\\ 1-\delta_s-\delta_{q_1}+\delta_{q_2^+} \ge 0\\ 1-\delta_s-\delta_{q_2}+\delta_{q_2^+} \ge 0\\ \delta_s-\delta_{q_1}+\delta_{q_1^+} \ge 0\\ \delta_s-\delta_{q_2}+\delta_{q_1^+} \ge 0\\ 0 \le \delta_s \le 1, 0 \le \delta_{q_1} \le 1, 0 \le \delta_{q_2} \le 1\\ 0 \le \delta_{q_1^+} \le 1, 0 \le \delta_{q_2^+} \le 1\end{gathered} \tag{2}$$

Here, each Boolean variable δ_P assumes the values 0 or 1 corresponding to the propositional statement *P* being *False* or *True*.

The general problem is to re-supply the capacitor while limiting the current drawn from the source. We will formulate an optimal control problem in discrete time as follows:

$$\min_{\delta_s} J,\ J = \alpha\left[q_N - \overline{q}\right]^2 + \frac{1}{N}\sum_{i=0}^{N-1} w_i^2 \tag{3}$$

With $i_L = 0$ an equilibrium point is $\overline{q} = EC$. We wish to steer the system from the initial state q_0 to near $\overline{q}$ over the time interval $t \in [0, T]$ along a trajectory that minimizes a quadratic performance index.

$$J = \alpha\left[q(T) - \overline{q}\right]^2 + \frac{1}{T}\int_0^T i^2\, dt \tag{4}$$

Subject to the dynamics and to the inequalities (2) we obtain (5). For specificity, we take $E = 1, C = 1, R = 1$.

$$q(t_{i+1}) = \delta_{q_1}\left(e^{-\Delta t} q(t_i) + \left[1 - e^{-\Delta t}\right]\right) + \left(1 - \delta_{q_1}\right) q(t_i) \tag{5}$$

We perform some further manipulations and introduce auxiliary variables that replace the nonlinear dynamics with linear dynamics and add linear inequality constraints in real (as opposed to integer) variables.

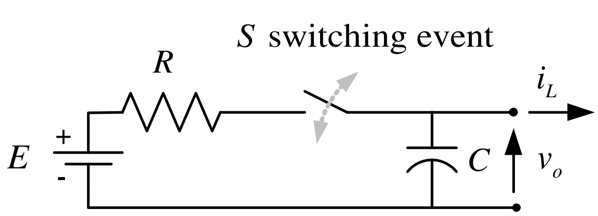

Figure 1 Power conditioning device.

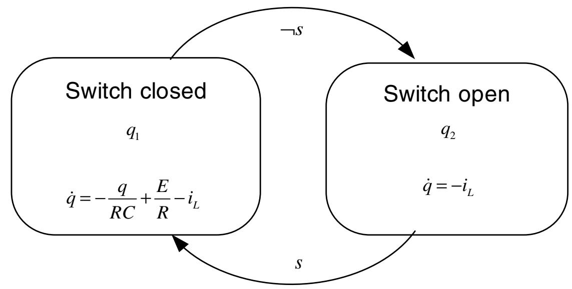

Figure 2 Open loop power conditioning system.

III. Results

We considered a variety of situations with different constant load currents, and various relative weightings between the costs associated with current and terminal time capacitor charge. With zero load current, the optimal switching strategy is as expected. For high current cost the switch remains open for all initial charge in the admissible range. On the other hand for low current cost, the switch remains closed. With nontrivial load, the switching strategy is more interesting although it still has a strong dependency on relative cost. For instance, with a current load of 0.1 amp, a time horizon of 2.5 sec, and terminal cost weight $\alpha = 0.25$, we obtain for $q \leq 1$ the switch is closed, for $q > 1$ the switch is open. If the weight is increased to $\alpha = 0.28$ the switch is closed for $q \leq 1$ and $q > 1.4$. It is open for $1 < q \leq 1.4$.

IV. Conclusion

In this panel contribution we discuss a *Mathematica*© based software tool, that converts a hybrid automaton model of a power system into either a mixed logical dynamic system' (MLD) or a 'dynamic mixed integer program' (DMIP). The concept is illustrated for the example of a power condition system. Results for the design of an optimal controller are presented.

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VI. Biographies

Harry G. Kwatny (M'70–SM'82–F'97) received the B.S.M.E. degree, the M.S. degree in aeronautics and astronautics, and the Ph.D. degree in electrical engineering from Drexel Institute of Technology, Philadelphia, PA, the Massachusetts Institute of Technology, Cambridge, and the University of Pennsylvania, Philadelphia, in 1961, 1962, and 1967, respectively. He joined the Department of Mechanical Engineering and Mechanics, Drexel University, Philadelphia, PA as an Assistant Professor in 1966, where he is currently the S. Herbert Raynes Professor of Mechanical Engineering. His research interests include modeling, analysis and control of nonlinear, parameter-dependent systems with specific applications to electric power systems and power plants, aircraft, spacecraft and ground vehicles. In recent years, he has focused on symbolic computing as a vehicle for bringing advances in nonlinear system theory into engineering practice. He has published over 100 papers in these areas. He is the coauthor with Prof. G. Blankenship (University of Maryland) of the book "Nonlinear Control and Analytical Mechanics: A Computational Approach" (Cambridge, MA: Birkhauser, 2000). He is also a coauthor of the software package TSi ProPac and a Mathematica package for nonlinear control system design and multibody mechanical system modeling.

Edoe F. Mensah received his B.A. degree in Mathematics from Rowan University in 1996 and a dual M.S. degree in Mathematics and in Electrical Engineering, from Drexel University in 1999. In 1999 he was also awarded the Albert J. Herr Scholarship for excellence in teaching. He has served as a mathematics instructor at Drexel University. He is currently pursuing his PhD in Mechanical Engineering at Drexel University. His research interests are hybrid systems, nonlinear control and robust control.

Dagmar Niebur (M '88) received her Diploma in Mathematics and Physics from the University of Dortmund, Germany in 1984. She received her Diploma in Computer Science (1987) and her Ph.D. in Electrical Engineering (1994) from the Swiss Federal Institute of Technology, Lausanne, (EPFL) Switzerland. In 1997 she joined Drexel University's Electrical and Computer Engineering Department where she is currently an associate professor. Prior to her engagement at Drexel she held research positions at the Jet Propulsion Laboratory and EPFL and a computer engineering position at the University of Lausanne. Her research focuses on intelligent information processing techniques for power system monitoring and control. In 2000, she received the NSF Career Award.

Carole Teolis is Vice President of Systems Engineering at Techno-Sciences, Inc. The Systems Engineering Group at Techno-Sciences, Inc. (TSi) has been involved in maintenance planning and equipment monitoring and prognostics since the early 90's. Dr. Teolis has lead efforts at TSi to develop advanced signal processing for engine health monitoring since 1998. She managed the installation of networked SCADA systems at the US Mint production coining facilities, developed adaptive coating mass control system for Bethlehem Steel, and was part of the team that developed the maintenance scheduling software system for OTIS Elevators North American Operations. Dr. Teolis interests are in signal processing, modeling and control systems design. Dr. Teolis earned a BS in Mathematics and a BS, MS and PhD degrees in Electrical Engineering from the University of Maryland, College Park.