Observability of Network-Delayed Multi-converter Power Systems

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***Abstract*—Power electronic converter systems have become a key feature of smart grid technologies due to the efforts to achieve increased levels of autonomy in power system operation. However, when devising control strategies for multi-converter power systems, nonlinearities-associated issues with these types of systems arise (e.g. bifurcations) and must be taken into account as even a slight variation of a system parameter could change the structure of the system. With the increasing need for remote control, investigation of converter system dynamics alone is insufficient. Remote control is achieved via a communication system that is used for information passing between sensors and controllers, which hence plays an important role on the overall performance of the power system. Therefore, a need arises for the study of the power system (plant) and the communication system (network) as a single entity as the performance of one is highly dependent on the other. The focus of this work is the development of a metric that will serve as an indicator of overall system status to the power system operator.**

I. INTRODUCTION

With the increase in automation of system functions within power electronics based power systems such as microgrids and shipboard power systems, the dependence on both executive and distributed controllers to maintain power quality, power flow, protection and performance of the system at acceptable levels is becoming more apparent. Traditional control approaches mainly based on linearized models of the system generation/distribution capacity will not be sufficient. One solution route is the development of nonlinear observer-based controllers through the incorporation of nonlinear power system dynamics. However, before this approach is used, first the system model inclusive of plant and network as one entity has to be developed and the concept of nonlinear observability addressed. The inherent control and communication infrastructure, as shown in Fig. 1, that layers on top of these new designs is of main significance when analyzing the system. Distributed remote controllers that oversee the operation of converters in a system are envisioned to accomplish control through the leveraging of communication networks. Thus, it becomes imperative that appropriate network control systems (NCS) are employed to ensure desired performance [1-4].

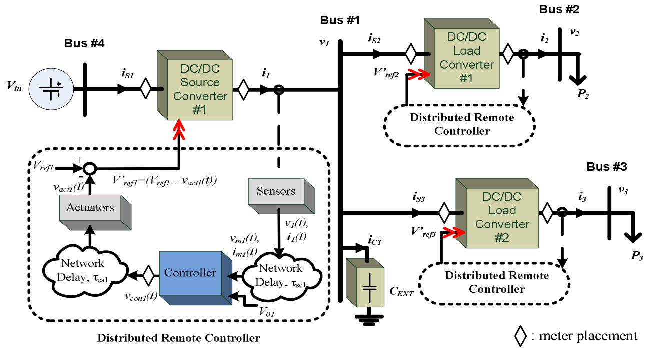

Figure 1. Multi-converter power system with distributed remote control

In the past the authors have investigated system modeling for multi-converter power systems and analyzed the communication delay effects in a single converter [5,6,9]. In this work, the goal is to investigate and illustrate the nonlinear behavior of multi-converter power systems and describe the existence of bounds for network parameters through the analysis of delay changes in the communication system and to address an observability formulation through the construction of an observability Jacobian inclusive of plant and communication dynamics.

II. PROBLEM FORMULATION

Monitoring and situational awareness through observations and control strategies in energy management systems provide solutions for an overall stable performance of a power system. However, these solutions are dependent on and sensitive to the implementation of sensors that account for parameter variations in the system and delays in the communication network. Therefore, the power system (plant) is to be studied as a single entity inclusive of the communication system (network) as the performance of one is highly dependent on the other. In the following subsections, modeling for the multi-converter power system with distributed remote control shown in Fig. 1 is presented and the concept of nonlinear observability addressed.

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A. *Multi-converter Power System Modeling*

Converters in Fig.1 are buck-boost converters with internal feedback control. Remote control for each converter is achieved through the implementation of distributed PI controllers as a first step towards the development of control strategies for the multi-converter system. For further details of the model the authors refer to their previous work in [5]. In this work the converter is assumed to operate in continuous conduction mode and the underlying stochastic nature of the communication network is ignored.

The authors have presented a Differential Algebraic Equations (DAE) model type in [5,6] where the multi-converter system was treated as a plant not inclusive of distributed remote controllers. In this paper, a system model that captures each converter and its distributed remote controller in the presence of network delay is presented. The generalized DAE model for parallel connected converters as shown in Fig. 1 is described by the following: the number of source converters, K, the number of load converters, M, and the total number of converters $T = K + M$. Converter # is denoted by i, where $i = 1,\dots, K$ for the source converters and $i = K + 1,\dots, T$ for load converters. Input voltage, inductor current, output voltage, measured output voltage, control voltage, actuation voltage, reference voltage, duty ratio, current feedback gain, voltage feedback gain, proportional gain of controller, integral gain of controller, desired output voltage, inductance, capacitance, load resistance, sensor-to-controller network time constant and controller-to-actuator network time constant of Converter #i, are denoted by V_{ini}, i_{Li}, v_i, v_{mi}, v_{coni}, v_{acti}, V_{refi}, d_i, k_{iLi}, k_{vi}, k_{Pi}, k_{Ii}, V_{0i}, L_i, C_i, R_i, τ_{sci} and τ_{cai}, respectively. Mechanical torque and speed of the generator connected to Converter #1 (source converter) are given by τ_1 and ω_1 respectively. An external capacitor connected to Bus #1 (Main DC Bus) for voltage regulation is given as C_{EXT}, the value of which was determined according to [7].

$$V'_{ref_i} = V_{ref_i} - v_{act_i} \tag{1}$$

$$d_i = V'_{ref_i} - k_{i_{L_i}} i_{L_i} - k_{v_i} v_i \tag{2}$$

$$\frac{di_{L_i}}{dt} = \begin{cases} \text{for } i = 1,\dots,K \\ \frac{1}{L_i}\left(-v_1 + V'_{ref_i} v_1 - k_{i_{L_i}} i_{L_i} v_1 - k_{v_i} v_1^2 + V_{in_i} V'_{ref_i} - k_{i_{L_i}} V_{in_i} i_{L_i} - k_{v_i} V_{in_i} v_1\right) \\ \text{for } i = K+1,\dots,T \\ \frac{1}{L_i}\left(-v_i + V'_{ref_i} v_i - k_{i_{L_i}} i_{L_i} v_i - k_{v_i} v_i^2 + v_1 V'_{ref_i} - k_{i_{L_i}} v_1 i_{L_i} - k_{v_i} v_1 v_i\right) \end{cases} \tag{3}$$

$$\frac{dv_i}{dt} = \begin{cases} \text{for } i = 1 \\ \frac{1}{C_{EXT}}\left[\sum_{j=1}^{K}\left(1-d_j\right) i_{L_j} - \sum_{k=K+1}^{T} d_k i_{L_k}\right] \\ \text{for } i = K+1,\dots,T \\ \frac{1}{C_i}\left(-V'_{ref_i} i_{L_i} + k_{i_{L_i}} i_{L_i}^2 + k_{v_i} v_i i_{L_i} + i_{L_i} - \frac{v_i}{R_i}\right) \end{cases} \tag{4}$$

$$\frac{dv_{m_i}}{dt} = \frac{1}{\tau_{sc_i}}\left(v_i - v_m\right) \tag{5}$$

$$\frac{dv_{con_i}}{dt} = \frac{-k_{P_i}}{\tau_{sc_i}}\left(v_i - v_{m_i}\right) + k_{I_i}\left(V_{0_i} - v_{m_i}\right) \tag{6}$$

$$\frac{dv_{act_i}}{dt} = \frac{1}{\tau_{ca_i}}\left(v_{con_i} - v_{act_i}\right) \tag{7}$$

$$0 = \tau_1\omega_1 + \sum_{j=2}^{K} V_{in_j} d_j i_{L_j} - \sum_{k=K+1}^{T} \frac{v_k^{\,2}}{R_k} \tag{8}$$

B. *Nonlinear Observability Formulation*

Observability requires that enough measurements exist and in power systems they should be distributed throughout the network, in such a way that state estimation is possible. Because knowledge of overall system behavior is critical even in situations of possible loss of measurements, state-estimation techniques should be developed. In order for these techniques to be valid, it is required that the system is observable. To address these concerns, one must develop the overall system model incorporating measurements.

The general model used to investigate power system dynamics is that of the Differential Algebraic Equations (DAE) type in (9) where $F(\cdot)$ is the set of functions relating to both the nonlinear dynamics and the nonlinear algebraic equations of the system model with z (dynamic and algebraic) state variables and N network parameters, p is a measurement vector, and u a set of independent control parameters (e.g. V_{ref} and V_0).

$$\begin{aligned} F(\dot{z}, z, N) &= u \\ p &= h(z, N) \end{aligned} \tag{9}$$

To derive the observability formulation, derivatives of the system (F) and observation (p) equations are taken as shown in (10) where s and r are the differentiation indices. The observability formulation is then given in terms of a general matrix form of the Jacobian in (11) where w are higher-order derivatives of $z(2\dots\,\sigma)$ and $\sigma = \max\{r, s+1\}$.

$$G = \begin{bmatrix} F(\dot{z}, z, N) \\ F_z(\dot{z}, z, N)\dot{z} + F_{\dot{z}}(\dot{z}, z, N)\ddot{z} \\ \vdots \\ \left(F(\dot{z}, z, N)\right)^{(s)} \end{bmatrix} = \begin{bmatrix} u \\ u^{(1)} \\ \vdots \\ u^{(s)} \end{bmatrix},\; H = \begin{bmatrix} h(z, N) \\ h_z(\dot{z}, z, N)\dot{z} \\ \vdots \\ h(\dot{z}, z, N)^{(r)} \end{bmatrix} = \begin{bmatrix} p \\ \dot{p} \\ \vdots \\ p^{(r)} \end{bmatrix} \tag{10}$$

$$J_O = \left[\begin{array}{c|c|c} G_z & G_{\dot{z}} & G_{z^{(w)}} \\ \hline H_z & H_{\dot{z}} & H_{z^{(w)}} \end{array}\right] = \left[\begin{array}{c|c|c|c|c} \frac{dF}{dz} & \frac{dF}{d\dot{z}} & 0 & 0 & 0 \\ \frac{dF^{(1)}}{dz} & \ddots & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \frac{dF^{(s)}}{dz} & \ddots & \frac{dF^{(1)}}{dz} & \frac{dF}{dz} & \frac{dF}{d\dot{z}} \\ \hline \frac{dh}{dz} & 0 & 0 & 0 & 0 \\ \frac{dh^{(1)}}{dz} & \ddots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 & 0 \\ \frac{dh^{(r)}}{dz} & \ddots & \frac{dh^{(1)}}{dz} & \frac{dh}{dz} & 0 \end{array}\right] \begin{array}{c} F \\ F^{(1)} \\ \vdots \\ F^{(s)} \\ h \\ h^{(1)} \\ \vdots \\ h^{(r)} \end{array} \tag{11}$$

$$\qquad z \qquad \dot{z} \qquad \cdot \qquad \cdot \qquad z^{(\sigma)}$$

The system is observable if the two conditions in (12) hold [8], where S is the neighborhood around the operating point given as the solution to (9). The placement, types and quantity of measurements in the system are some of the concerns that arise related to this observability determination. The goal therefore is to construct J_O for the multi-converrter power system model case, and then check the observability conditions of (12).

$$1: rank(J_O) = n + rank\begin{bmatrix} G_{\dot{z}} & G_{z^{(w)}} \\ H_{\dot{z}} & H_{z^{(w)}} \end{bmatrix} \quad (12)$$

$$2: rank(J_O) \text{ is constant rank on S}$$

III. Analysis, Simulation and Results

A. Critical Round-trip Time Delay

The goal of the following study was to investigate the critical round-trip time delay ($RTT_{critical}$) of a multi-converter power system. The critical round-trip time delay for a single converter is the maximum round-trip time delay in the communication path the system can withstand while maintaining stability and is computed as shown in (13) [9].

$$RTT_{critical} = (\tau_{sc} + \tau_{ca})_{critical} \quad (13)$$

A set of parameters that must be determined before computing $RTT_{critical}$ is the PI controller gain pairs (k_{Pi}, k_{Ii}) of each single converter. These gains were obtained by tuning the controller so that it accounts for an average network delay of $\tau = 0.001$ s and $\tau = 0.050$ s taking only local state behavior into consideration as if the converter is operating in stand-alone mode. This was accomplished via the PID Controller Block in Simulink. The proportional and integral gains obtained for the different network delays are shown in Table I. The operating point of the system, x^{ss} is given in Table II. The numerical values of the states corresponding to the operating point are not dependent on the network delay value. However, the stability properties of this equilibrium point may change under varying delay which will be indicated by the metric $RTT_{critical}$.

TABLE I. PROPORTIONAL AND INTEGRAL GAINS OF EACH PI CONTROLLER TUNED FOR AN AVERAGE NETWORK DELAY OF τ = 0.001 s AND τ = 0.050 s

Converter #*i*	**PI controller gains accounting for an average delay of τ (s)**			
	τ = 0.001 s		**τ = 0.050 s**	
	k_{Pi} (V^{-1})	k_{Ii} ($V^{-1}s^{-1}$)	k_{Pi} (V^{-1})	k_{Ii} ($V^{-1}s^{-1}$)
i = 1	-0.0987	-15.1087	-0.1219	-0.7959
i = 2	-0.0554	-14.0945	-0.0998	-0.7036
i = 3	-0.0468	-6.1301	-0.0416	-0.3604

TABLE II. STEADY-STATE OPERATING POINT OF MULTI-CONVERTER SYSTEM

Converter #*i*	**Steady-state operating point**				
	i_{Li}^{ss} (A)	v_i^{ss} (V)	v_{mi}^{ss} (V)	v_{coni}^{ss}	v_{acti}^{ss}
i = 1	18.3333	100.0000	100.0000	0	0
i = 2	11.2500	80.0000	80.0000	0	0
i = 3	10.5556	90.0000	90.0000	0	0

For simplicity, it is assumed that $\tau_{sci} = \tau_{cai} = \tau_i$ in the multi-converter system. Therefore, now (13) can be rewritten as,

$$RTT_{critical} = 2(\tau_i)_{critical} \quad (14)$$

In these studies, several different cases of network time constants were considered as shown in Table III. In order to determine $RTT_{critical}$ for each converter in stand-alone mode, first, the nonlinear system of equations is linearized around its operating point. The network time constant breaches the critical round-trip time delay when the eigenvalues of the Jacobian matrix computed for each set of parameters crossover to the Right-Half Plane (RHP) from the Left-Half Plane (LHP). The authors acknowledge that changes in the time constants mentioned above might not affect the system in the same manner; hence, the need to study the effects of each network time constant on the system separately. However, the focus of this work is the effects of system topology on the aggregate quantity that is the critical round-trip-time delay of the converter(s) of interest and consequently the stability properties of the system.

TABLE III. PERFORMED CASE STUDIES

Case #	**Converter(s) of Interest**	**Procedure**
1	1	Vary τ_1 while $\tau_2 = \tau_3 = 1$ ms
2	2	Vary τ_2 while $\tau_1 = \tau_3 = 1$ ms
3	3	Vary τ_3 while $\tau_1 = \tau_2 = 1$ ms
4	1, 2 and 3 (full multi-converter system)	Vary τ_1, τ_2 and τ_3 so that $\tau_1 = \tau_2 = \tau_3$

In Table IV, the critical round-trip time delay for each of the cases described in Table III is shown. Next to each of these critical round-trip time delay values, the critical round-trip time delay for the converter of interest in stand-alone operation is shown within parenthesis. From Table V, it was observed that accounting for a higher network delay during the controller design process, results in a higher $RTT_{critical}$. It can also be observed that cases 1 and 4 result in the two lowest $RTT_{critical}$ values. A higher delay in the communication network associated with the source converter leads to instability faster. However, further investigation on how each delay parameter affects the operation of each converter will be necessary when taking corrective measures.

TABLE IV. CRITICAL ROUND-TRIP TIME DELAY FOR CONVERTERS OF INTEREST IN MULTI-CONVERTER POWER SYSTEM

Case #	**$RTT_{critical}$ (s) for converter of interest and power mismatch when PI controller tuned accounting for an average delay of τ (s)**			
	τ = 0.001 s		**τ = 0.050 s**	
	$RTT_{critical}$ (s)	**ΔP (W)**	**$RTT_{critical}$ (s)**	**ΔP (W)**
1	0.002 *(0.022)*	$-4.89\text{x}10^{-12}$	0.010 *(0.886)*	$-3.52\ \text{x}10^{-12}$
2	0.012 *(0.014)*	$-3.18\ \text{x}10^{-12}$	0.633 *(0.800)*	$-1.82\ \text{x}10^{-12}$
3	0.014 *(0.018)*	$-9.09\ \text{x}10^{-13}$	0.566 *(0.678)*	$-6.82\ \text{x}10^{-13}$
4	0.002 *(N/A)*	$-4.89\ \text{x}10^{-12}$	0.018 *(N/A)*	$1.14\ \text{x}10^{-13}$

B. Observability Jacobian: Construction and Rank Evaluation

To illustrate the construction of the observability Jacobian for the multi-converter system of Fig. 1 where $z \in \mathbb{R}^n$, and $h \in \mathbb{R}^m$, the system of equations described by (3) – (8) is used and the differentiation indices set to $r = s = 1$. It is assumed that measurements described by (15) are available in the system. These are converters' input and output currents, i_{Sx} and i_x, output voltage, v_x, output power, P_x, and voltage control signal of distributed remote controller, v_{conx}. This means that for each converter in the system, up to five measurements can be taken and incorporated into the construction of the system's observability Jacobian.

$$\begin{aligned} &p_1 : i_{Sx} = d_x i_{L_x} \\ &p_2 : i_x = (1-d_x) i_{L_x} \\ &p_3 : v_x^{out} = v_x \\ &p_4 : P_x = v_x i_X = v_x (1-d_x) i_{L_x} \\ &p_5 : v_{con}^{out} = v_{con} \end{aligned} \quad (15)$$

Following the general form of (11), dF/dz is a n x n matrix of partial derivatives related to the nonlinear algebraic and dynamic equations of the system model, and $dF/d\dot{z}$ is given by (16) where a and b are the number of dynamic and algebraic state variables respectively.

$$\frac{dF}{d\dot{z}} = \begin{bmatrix} \boldsymbol{I}^{a\times a} & \boldsymbol{0}^{a\times b} \\ \boldsymbol{0}^{b\times a} & \boldsymbol{0}^{b\times b} \end{bmatrix}^{n\times n} \tag{16}$$

Examining the measurement equations in (15), sub-matrices H_z, $H_{\dot{z}}$, and $H_{z^{(w)}}$ in (11) are dependent on the specific types of measurements available and dh/dz is a m x n matrix. If the observability Jacobian is evaluated around an operating point, $dF^{(1)}/dz \cdots dF^{(s)}/dz$ and $dh^{(1)}/dz \cdots dh^{(r)}/dz$ are null sub-matrices. The observability Jacobian in (11) has a size of $[(r+1)m + (s+1)n]$ x $[(\sigma+1)n]$ and its dimensionality is not only affected by the differentiation indices s and r, and the number of system measurements but by the selected system (plant) model.

After the observability Jacobian is constructed, the conditions of (12) have to be checked. Rank evaluation by checking condition #1 of (12) and by direct calculation of the overall Jacobian rank is presented in Table V. Overall Jacobian matrix and sub-matrices dimensions are also given in Table V. Parameter values for internal control feedback gains and reference voltage for converter #i in the system are: for i = 1, k_v = 0.25, k_{iL} = -0.01, and V_{ref} = 4.0379; for i = 2, k_v = 0.30, k_{iL} = -0.02, and V_{ref} = 2.2194; and for i = 3, k_v = 0.20, k_{iL} = -0.015, and V_{ref} = 1.2348. Distributed remote controller PI gains are those in Table I when an average delay of τ = 0.001s is accounted for. Four measurement sets were investigated. Measurement set #1 includes output powers of source converter and load converters (i.e., p_1, p_2, p_3) ; set #2 includes measurements in set #1 and output currents and output voltages (i.e., i_1, v_1, i_2, v_2, i_3, v_3); set #3 includes set #2 and converters' input currents (i.e., i_{S1}, i_{S2}, i_{S3}); and set #4 includes set #3 and remote controllers' voltage control signal measurements (i.e., v_{con1}, v_{con2}, v_{con3}). It is observed that depending on the measurement set used, additional rows are added to the overall dimensionality of J_o. The approach of constructing the observability Jacobian is not altered by the measurement types or location but it enhances the rank of the overall matrix when checking conditions described by (12).

TABLE V. RANK EVALUATION AND DIMENSIONALITY OF J_O AND SUB-MATRICES

	Dimensionality			Rank	
Measurement	J_o	$G_z, G_{\dot{z}}, G_{\ddot{z}}$	$H_z, H_{\dot{z}}, H_{\ddot{z}}$	From (12)	Rank(J_o)
Set #1	38×48	32×16	6×16	47	38
Set #2	50×48		18×16		42
Set #3	56×48		24×16		44
Set #4	62×48		30×16		47

The dimensionality of the overall observability Jacobian is affected by the number of available system measurements, in particular the size of the measurement sub-matrices. As seen in Table V, the size of sub-matrices associated to the plant are unchanged as more measurements were included. Measurement set #4 is the only set that satisfies condition #1 of (12) as the evaluation of the Jacobian with this measurement set is equal to the rank obtained when it is computed from the overall Jacobian. Once the conditions in (12) are satisfied, the observability Jacobian can be utilized to evaluate when the system will lose or is close to losing observability.

IV. CONCLUSIONS

This work focuses on modeling and analysis of information-embedded multi-converter power systems. The effect of network time delay on system stability as well as observability was investigated by computation of the metric, critical round-trip time and the rank of the observability Jacobian. The inclusion of time delays in controller tuning improves the critical round-trip time delay as expected. Studying the system behavior under different network delay conditions provides insight into communication paths, the failure or congestion of which can cause system-wide instability. A sensitivity study of how each communication path affects each subsystem in the multi-converter power system will enable operator to device appropriate distributed or supervisory control strategies as corrective measures in the event of communication failure.

The dimensionality of the observability Jacobian is affected by the system model, the differentiation indices s and r, and the system measurements. For the specific system model presented in this paper, the rank observability condition was examined as the number of measurements was incremented. Besides plant measurements, distributed remote controller measurements had to be included in the construction of the observability Jacobian to satisfy a rank condition.

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