A Game-theoretic Approach to Power Management in MIMO-OFDM Ad Hoc Networks

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Dedications

For Mom and Dad

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Abstract

A Game-theoretic Approach to Power Management in MIMO-OFDM Ad Hoc Networks

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With the increasing demand for wireless services, the efficient use of spectral resources is of great importance. MIMO-OFDM communication systems hold great promise in using radio spectrum efficiently while power control will improve energy efficiency. Existing approaches such as multiuser water-filling and gradient projection assign a fixed transmit power to each link and each transmitter node allocates power among different antennas in order to optimize the link capacity or sum data rate. If bad channel conditions exist in some communicating links, these methods are not energy efficient.

We propose a new technique for power management and interference reduction based upon a game theoretic approach. Utility functions are designed and power allocation in each link is built into a non-cooperative game. To avoid unnecessary power transmission under poor channel conditions, a mechanism of shutting down inefficient links is integrated into the game theoretic approach. Two kinds of link shut-down mechanism are presented in this dissertation. The first one is called hard shut-down, because once the transmit node decides to shut down, the node will not resume transmission no matter how the interfering channels change. The other mechanism is called soft shut-down, in which the transmit power is related to the pricing factor of that link and the interference it is exposed to. With this mechanism, the transmit power can change adaptively in response to the condition of interference.

We also investigate the problem of subcarrier assignment and power distribution among multiple antennas for point-to-point links in a network without base stations. A subcarrier assignment scheme is proposed which selects a set of subcarriers for each link so that high data rate can be achieved and co-channel interference can be mitigated. The power management in a MIMO-OFDM ad hoc network is also built into a non-cooperative game

in which each link calculates its optimal power allocation vector in order to maximize the net utility. The designed utility function facilitates subcarrier assignment schemes by using a tunable pricing factor, which helps a link to admit or drop subcarriers in a soft and adaptive fashion.

1. Introduction

1.1 Introduction to MIMO

Demands for capacity in wireless communications, driven by cellular mobile, Internet and multimedia services have been rapidly increasing worldwide. On the other hand, the available radio spectrum is limited and the communication capacity needs cannot be met without a significant increase in communication spectral efficiency. Significant further advances in spectral efficiency are available through increasing the number of antennas at both the transmitter and the receiver [13][34]. The benefits of exploiting MIMO can be categorized by the following :

Array gain

Array gain refers to the average increase in the SNR at the receiver that arises from the coherent combining effect of multiple antennas at the receiver or transmitter or both. The average increase in signal power at the receiver is proportional to the number of receive antennas.

Diversity gain

Signal power in a wireless channel fluctuates. When the signal power drops significantly, the channel is said to be in a fade. Diversity is used in wireless channels to combat fading. Utilization of diversity in MIMO channels requires antenna diversity at both receive and transmit side. The diversity order is equal to the product of the number of transmit and receive antennas, if the channel between each transmit-receive antenna pair fades independently.

Spatial multiplexing (SM)

SM offers a linear (in the number of transmit-receive antenna pairs or $\min(M_r, M_t)$) increase in the transmission rate for the same bandwidth and with no additional power consumption.

Interference reduction

Co-channel interferece arises due to frequency reuse in wireless channels. When multiple antennas are used, the difference between the spatial signatures of the desired signal and co-channel signals can be exploited to reduce the interference. This operation is done at the receiver side and it requires knowledge of the channel of the desired signal. Interference reduction can also be done at the transmitter, where the goal is to minimize the interference energy sent towards the co-channel users while delivering the signal to the desired user.

1.2 Introduction to OFDM

Orthogonal Frequency Division Multiplexing (OFDM) has become a popular technique for transmission of signals over wireless channels. OFDM is a multicarrier transmission technique, which divides the available spectrum into many subcarriers, each one being modulated by a low-rate data stream. It efficiently uses the spectrum by spacing the channels closer together. This is achieved by making all the subcarriers orthogonal to one another, preventing interference between closely spaced carriers. OFDM converts a frequency-selective channel into a parallel collection of frequency flat subchannels. A key advantage of OFDM systems is their inherent capability to allow adaptive resource allocation where the level of modulation, the number of bits loaded and the transmit power in each subcarrier, can be selected to increase the data rate or reduce the required transmit power [6]. For instance, if knowledge of the channel is available at the transmitter, then the OFDM transmitter can adapt its signaling strategy to match the channel and the ideal water-filling capacity of a frequency-selective channel can be approached [33].

As it is well known that the MIMO technique utilizes spectrum efficiently and enhances energy efficiency, the hybrid design of MIMO and OFDM further enables the diversities from spatial, temporal and spectral domains, which can lead to significant improvement in system performance.

1.3 Power Management in Wireless Communication Systems

Power control plays a key role in improving energy efficiency of wireless communication systems. It is important to identify ways to use less power while maintaining a certain quality of service (QoS). There has been a considerable amount of research on power management in wireless systems. In [36, 41], power control algorithms were developed for cellular systems. Power control has also been studied with a combination of multiuser detection, beamforming and adaptive modulation[38, 22]. In [21, 32] adaptive algorithms were developed to improve system performance by controlling power allocation and data rate. As the use of MIMO technology in ad hoc networks grows, MIMO interference systems have attracted a great deal of attention. [40, 2] studied the interactions and capacity dependencies of MIMO interference systems and [37, 26] explored methods for power management and interference avoidance in MIMO systems.

In recent years there has been a growing interest in applying game theory to study wireless systems. [29, 16] used game theory to investigate power control and rate control for wireless data. Their work studied mobile cellular networks with transmitter and receiver pairs using Single-Input-Single-Output (SISO) antennas. The analysis was based on the Signal to Interference plus Noise Ratio (SINR), which was a function of the transmit power of each individual transmitter. Since SISO antenna system was used, the controlling variable of each user was its transmit power which was a scalar.

Hicks provided a game theory perspective on interference avoidance in [17]. A synchronous interference avoidance (IA) scheme was modeled as a potential game. In addition, when the IA system's signal environment is levelable, the noisy best response iteration almost surely converges for a game in which links independently choose from a set of metrics given in that paper.

A game-theoretic approach to study power allocation in MIMO channels was developed in [25]. This paper considers the case in which even the channel statistics are not available at the transmitter, obtaining a robust solution under channel uncertainty by formulating the problem within a game-theoretic framework. The payoff function of the game is the mutual information and the players are the transmitter and a malicious nature. The problem turns out to be the characterization of the capacity of a compound channel which is mathematically formulated as a maximin problem. The uniform power allocation is obtained as a robust solution.

In practical wireless communication systems, transmitters try to obtain channel state information (CSI). It can be acquired either via a feedback channel or the application of the channel reciprocity property to previous receive channel measurements when the transmit and receive channels are sufficiently correlated. When CSI is available at the transmitter, the optimal power allocation that achieves capacity is well known [9]. In such a case, capacity is achieved by adapting the transmitted signal to the specific channel realization. To be more specific, the channel matrix is diagonalized and the set of constituent subchannels or eigenmodes is obtained. The optimal signaling directions are the eigenmodes of the channel matrix and power is allocated preferably to those eigenmodes with higher eigenvalues. In this dissertation, we will assume transmitters instantly know channel conditions due to a no-delay feedback mechanism. With this information, the transmitters find out their power allocations based on water-filling solutions. These solutions correspond to maximizing channel mutual information by using the maximum transmit power available.

In a wireless ad hoc network with multiple co-channel links, the transmitter in a link not only sends data to its designated receiver, but also causes interference to other links. Some links may have excellent channel conditions and accordingly support high data rate. However, if the capacity of a link is more than enough to maintain a certain level of QoS, reducing the capacity by decreasing transmit power will mitigate the interference sent to other links, i.e., it is not necessary to transmit the maximum amount of power. In contrast, some links may have the transmitter and the receiver nodes far apart with poor channel conditions, thus the data rates that these links can support are low even the maximum power is transmitted. Those low-rate links are not only useless for data transmission but also may bring down the data rates of other links due to generated interference. To counter interference, other links may also increase their transmit power, which results in low energy efficiency across the network. To avoid this negative effect, it is advisable to shut down low-efficiency links. In order to accommodate the two scenarios described above, we can assign each link a utility function, which has an intrinsic power control property. By maximizing its utility function, each link tries to reach a high data rate without necessarily transmitting the highest power. If even a minimum data rate cannot be supported by a link, the link shuts off in order to save power and reduce interference. In a wireless ad hoc network, there is no central controller to determine the strategy of resource allocation for each link. Instead, a link acquires the information about channel state as well as interference, then makes its own decision how to allocate resource. This kind of interaction among wireless links can be modeled as a non-cooperative game [15], where each link attempts to selfishly maximize its utility. If the utility function is well-designed with the power control property described before, there will be implicit coordination among wireless links so that metrics reflecting social preference such as energy efficiency or sum data rate of the network can be improved.

1.4 Outline of Dissertation

In general terms, this dissertation focuses on power management in MIMO-OFDM ad hoc networks. In Chapter 2, we consider a stationary MIMO ad hoc network, where each transceiver pair is hindered by cochannel interference coming from other transceiver pairs operating in the same frequency band. We investigate optimum signaling for MIMO interference systems with feedback in a realistic ad hoc network environment and study how power control improves energy efficiency by using a game theoretic approach. Computational electromagnetic simulations [10] are used to study the effect of interference on a network composed of multiple, cochannel MIMO links. A game theoretic approach for power control is proposed where we construct a non-cooperative power control game and show how to design a utility function suitable for MIMO ad hoc networks. A mechanism of shutting down inefficient links is included in the power management. Asymptotic behaviors of that power control game are investigated as well.

In Chapter 3, we point out that the previous chapter contains a hard link shut-down mechanism, and the game analysis based on that hard mechanism is only applicable to those viable links. In real situations, the channel condition of a link may change from time to time and the interference that link experiences also fluctuates, so it will be beneficial to set up a mechanism which allows a link to adaptively control its transmit power. This mechanism is expected to turn off a link if its data rate is too low, but allows a link to transmit if channel condition has improved, therefore it is called a soft shut-down mechanism. In this chapter, we carefully design a new utility function which is the link data rate minus the scaled transmit power. The second term is considered a price charged for using resource. We will prove in theory that a link is actually shut down if the pricing factor is properly chosen. The power management is again built into a non-cooperative game and we will investigate the existence of Nash equilibrium.

In Chapter 4, power management in a game theoretic approach is extended to MIMO-OFDM ad hoc networks, where there are multiple point-to-point wireless links. Since every link works in the same frequency band, co-channel interference (CCI) is a key factor to limit the data rate of each link. Therefore, resource allocation has to be done with serious considerations in interfering links. OFDM systems provide a degree of freedom to allocate power in each subcarrier. If different links use a different subset of available subcarriers, interference experienced by each link may be mitigated due to the fact that a particular subcarrier may not be used by all links. In this chapter, we investigate joint subcarrier assignment and power allocation for MIMO-OFDM ad hoc networks and a subcarrier assignment scheme is proposed. The power management in a MIMO-OFDM ad hoc network is also built into a non-cooperative game and it is shown that the designed utility function facilitates subcarrier assignment schemes by using a tunable pricing factor, which helps a link to admit or drop subcarriers in a soft and adaptive fashion.

1.5 Notations

We will try to remain consistent with notations throughout this dissertation. However, if inconsistence does occur, the notation should be clear from context or we will define it immediately. In this dissertation notations are used as follows: Lower case and bold face letters denote vectors, e.g., $\mathbf{x}, \mathbf{q}$, whose elements may be scalars, vectors or matrices. Upper case and bold face letters denote matrices or the set of matrices, e.g., $\mathbf{H}, \mathbf{Q}$. $E\{\cdot\}$ denotes expectation, $\nabla f(\cdot)$ denotes the gradient of $f(\cdot)$. For a matrix $\mathbf{A}$, $\mathbf{A}^T$ denotes transpose, $\mathbf{A}^\dagger$ denotes the conjugate transpose, $\mathrm{Tr}(\mathbf{A})$ denotes the trace, $\det(\mathbf{A})$ denotes the determinant and $\mathbf{A} \geq \mathbf{0}$ denotes that $\mathbf{A}$ is positive semidefinite. A n-dimensional identity matrix is denoted by $\mathbf{I}_n$ or $\mathbf{I}$ in case it is self-evident. $\mathbb{R}_+^n$ denotes the n dimensional nonnegative orthant.

2. Power Management in MIMO Ad Hoc Networks - A Game Theoretic Approach

With the increasing demand for wireless services, the efficient use of spectral resources is of great importance. Multiple Input Multiple Output (MIMO) communication systems hold great promise in using radio spectrum efficiently [14] while power control will improve energy efficiency. In applications like wireless ad hoc networks, battery life is the largest constraint in designing algorithms [7]. Therefore, it is important that power allocation be managed effectively by identifying ways to use less power while maintaining a certain quality of service (QoS).

In this chapter, we consider a stationary MIMO ad hoc network, where each transceiver pair is hindered by cochannel interference coming from other transceiver pairs operating in the same frequency band. It is known that minimizing interference using power control increases capacity and extends battery life for cellular systems [29]. We investigate optimum signaling for MIMO interference systems with feedback in a realistic ad hoc network environment and study how power control improves energy efficiency by using a game theoretic approach. We use computational electromagnetic simulations [10] to study the effect of interference on a network composed of multiple, cochannel MIMO links. These simulations, given a network topology and environment, calculate the received electromagnetic fields due to all of the multipath rays between every transmitter and every receiver. The simulations are performed using the software system FASANT, which has been used as a tool in system planning and has been validated using urban propagation measurements [5].

This chapter is organized as follows. Section 2.1 introduces the system model and formulates the optimization problem. Existing techniques from the literature are also introduced as a basis for comparison. In Section 2.3, a game theoretic approach for power control is proposed where we construct a non-cooperative power control game and show

how to design a utility function suitable for MIMO ad hoc networks. Asymptotic behaviors of that power control game are investigated as well. Simulation results with all methods are given and discussed in Section 2.4. Section 2.5 provides the summary.

2.1 Problem Formulation

Consider an ad hoc network with a set of links denoted by $\mathbb{L} = \{1, 2, ..., L\}$, where each link undergoes cochannel interference from the other $L-1$ links. Each node uses N_t transmit antennas and N_r receive antennas. The channel between the receive antennas of link l and the transmit antennas of link j is denoted by $\mathbf{H}_{l,j} \in C^{N_r \times N_t}$. For all l of the L links, the transmitted signal vector, $\mathbf{x}_l \in C^{N_t \times 1}$ has covariance matrix $\mathbf{Q}_l = E\{\mathbf{x}_l \mathbf{x}_l^\dagger\}$ and the receiver array performs independent single-user detection. The received baseband signal of link l, $\mathbf{y}_l \in C^{N_r \times 1}$, is given by

$$\mathbf{y}_l = \mathbf{H}_{l,l}\mathbf{x}_l + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{x}_j + \mathbf{n}_l \quad (2.1)$$

where $\mathbf{n}_l \in C^{N_r \times 1}$ is the noise vector with independent complex Gaussian entries. We also call $\mathbf{Q}_l$ a power allocation matrix with the transmit power for link l given by $\mathrm{Tr}(\mathbf{Q}_l)$. The instantaneous data rate of link l is obtained as [9]

$$C_l(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \log_2 \det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^\dagger \mathbf{R}_l^{-1}) \quad (2.2)$$

where $\mathbf{R}_l = \mathbf{I} + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{Q}_j\mathbf{H}_{l,j}^\dagger$ is the interference-plus-noise matrix of link l. The channel matrix $\mathbf{H}_{l,j}$ and $\mathbf{R}_l$ are calculated by our computational electromagnetic simulations. In addition, due to an assumed no-delay channel feedback mechanism, the transmitters instantly know channel conditions.

Each transmitter adjusts its power allocation in an effort to maximize its data rate. Power adjustment can be done in two ways. In the first technique, for a fixed transmit power of each node, the power is allotted among the multiple transmit antennas to achieve capacity maximization. The second technique allows power control for transmitters, i.e., the transmit power for a certain link l, $p_l = \mathrm{Tr}(\mathbf{Q}_l)$, can be adjusted. Using this power control, the transmitter can follow two courses of action: it can change the total power allotted to the link and it can also allot this power in different ways among the multiple antennas of the link.

For an ad hoc network, the mutual information of the L-link system given all channel matrices $\mathbf{H}_{1,1}, \ldots, \mathbf{H}_{L,L}$ is

$$C(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \sum_{l=1}^{L} \log_2 \det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}) \tag{2.3}$$

For a metric of energy efficiency, we use the ratio of the system capacity over the total power consumption. This metric corresponds to the amount of achievable capacity per unit energy.

$$\lambda = \frac{\sum_{l=1}^{L} C_l}{\sum_{l=1}^{L} p_l} \tag{2.4}$$

2.2 Existing techniques to calculate system mutual information

Different algorithms have been developed to calculate system mutual information. The most commonly used ones will be introduced below.

2.2.1 Independent Water-filling

This approach is optimum for the non-interference situation, where the transmitter pretends that there is no interference from other links. For each link, it is essentially a singe-user system and the mutual information is given by

$$C(\mathbf{Q}) = \log_2 \det(\mathbf{I} + \mathbf{HQH}^{\dagger}) \tag{2.5}$$

and the power allocation matrix is subject to $\mathrm{Tr}(\mathbf{Q}) \leq \bar{p}$ and $\mathbf{Q} \geq \mathbf{0}$, where $\bar{p}$ is the maximum transmit power.

This single-link optimization problem has a well-known water-filling solution [9]. Let

$\mathbf{H}^\dagger\mathbf{H} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^\dagger$ be an eigenvalue decomposition of $\mathbf{H}^\dagger\mathbf{H}$, with $\mathbf{V}$ unitary and $\mathbf{\Sigma}$ diagonal, then the optimum power allocation is given as $\mathbf{Q} = \mathbf{V}(\mu\mathbf{I} - \mathbf{\Sigma}^{-1})^+\mathbf{V}^\dagger$ where μ is called the water level and is chosen to satisfy $\mathrm{Tr}(\mu\mathbf{I} - \mathbf{\Sigma}^{-1})^+) = \bar{p}$. This solution indicates that the optimal signaling directions are the eigenmodes of the channel matrix and power is allocated preferably to those eigenmodes with higher eigenvalues. Fig.2.1 illustrates how power is allocated relative to the eigenvalues and the water level, where σ_i's are the diagonal elements of $\mathbf{\Sigma}$ and z_i's are the diagonal elements of $(\mu\mathbf{I} - \mathbf{\Sigma}^{-1})^+$. For a network system, if all links are coordinated such that only one link is transmitting at a time, that link does not undergo interference and therefore its mutual information can be maximized with an independent water-filling solution. Another interesting scenario occurs in an OFDM-based MIMO system, where we assume perfect frequency synchronization is in place. If all subcarriers are partitioned into non-overlapping subsets and each link in the network uses a subset of the subcarriers, this system is in fact interference-free as transmission on one frequency does not cause interference to another.

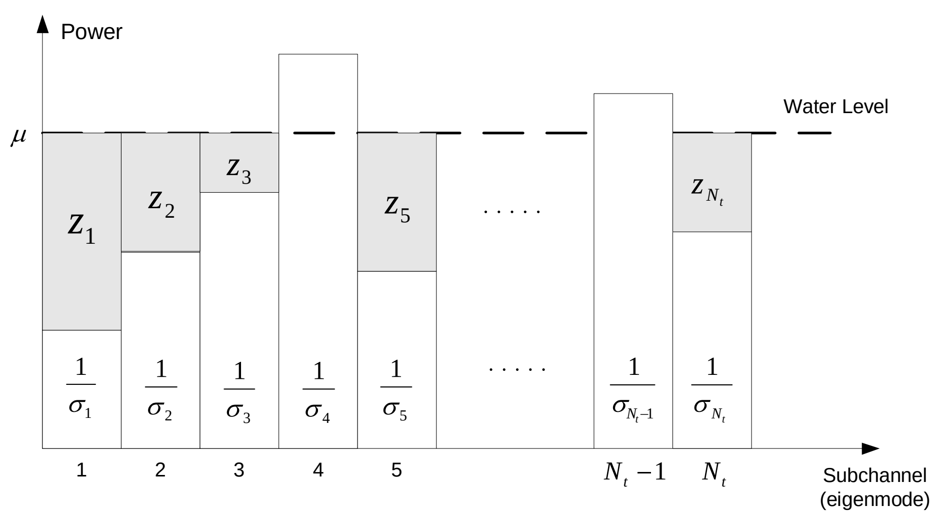

Figure 2.1: Water-filling power allocation $z_i = (\mu - \sigma_i^{-1})^+$

.

2.2.2 Multiuser Water-filling

In a network with multiple interfering links, the mutual information of a link is related to the noise and interference matrix, which varies with the transmitter correlation matrices of the interfering nodes. A change in the power allocation matrix of one link induces a change in the optimum power allocation matrices of the other co-channel links. Yu proposed an iterative water-filling algorithm to compute the optimal input distribution of a Gaussian multiple access channel with multiple vector inputs and a single vector output so that the sum data rate of the channel is maximized [40]. In this method, each transmitter is assumed to know its own channel information $\mathbf{H}_{l,l}$ as well as the noise and interference matrix $\mathbf{R}_l$. It is shown that the sum data rate problem can be broken down into single-user problems so that the power allocation matrices can be found iteratively. Specifically, at each iteration, transmitter l tries to solve the following optimization problem.

$$\begin{aligned} \max_{\mathbf{Q}_l} \quad & \log_2 \det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}) \\ s.t. \quad & \mathrm{Tr}(\mathbf{Q}_l) \le \bar{p}_l \\ & \mathbf{Q}_l \ge \mathbf{0} \end{aligned} \tag{2.6}$$

This problem has a similar water-filling solution to the interference-free case. Mathematically, using the conversion

$$\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l} = \mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-\frac{1}{2}}\mathbf{R}_l^{-\frac{1}{2}}\mathbf{H}_{l,l} = (\mathbf{R}_l^{-\frac{1}{2}}\mathbf{H}_{l,l})^{\dagger}(\mathbf{R}_l^{-\frac{1}{2}}\mathbf{H}_{l,l}) = \tilde{\mathbf{H}}_{l,l}^{\dagger}\tilde{\mathbf{H}}_{l,l} = \mathbf{U}_l\mathbf{\Sigma}_l\mathbf{U}_l^{\dagger} \tag{2.7}$$

with $\mathbf{U}_l$ unitary and $\mathbf{\Sigma}_l$ diagonal, link l employs the power allocation $\mathbf{Q}_l = \mathbf{U}_l(\mu_l\mathbf{I} - \mathbf{\Sigma}_l^{-1})^{+}\mathbf{U}_l^{\dagger}$ to maximize instantaneous mutual information and μ_l is chosen so that $\mathrm{Tr}(\mu_l\mathbf{I} - \mathbf{\Sigma}_l^{-1})^{+}) = \bar{p}_l$. Comparing the multiuser water-filling with the independent water-filling, we can see that they are essentially the same if in the former the substitution $\tilde{\mathbf{H}}_{l,l} = \mathbf{R}_l^{-\frac{1}{2}}\mathbf{H}_{l,l}$ is applied. This operation is called "spatially whitening transform"[11], since it simplifies an interference plus noise MIMO channel into a noise-only MIMO channel as far as the mutual information is concerned. In [40] the iterative water-filling algorithm always converges to the sum rate given the output is a single vector. However, for an ad hoc network, each links tries to maximize its own mutual inforamtion and the system mutual information is the sum data rate of all links, where the number of output vectors is the same as the number of links, thus convergence is not guaranteed. In fact, it is pointed out in [37] that this algorithm does not always converge in the MIMO ad hoc network scenario.

2.2.3 Gradient Projection Method

In the multiuser water-filling method every link non-cooperatively competes with others so as to achieve the highest data rate individually. A more complex problem is to maximize $C(\mathbf{Q}_1, ..., \mathbf{Q}_L)$, which is the sum data rate of the network. To achieve this, the transmitters need to cooperate in a certain way when deciding their covariance matrices. In [37], the gradient projection (GP) method [1] which is widely used as an unconstrained steepest descent method, is extended to convex constrained problems in order to solve the sum data rate problem. The main idea is: Each link tries to maximize $C(\mathbf{Q}_1, ..., \mathbf{Q}_L)$ iteratively. When link l is optimizing the objective function, it assumes the power allocation matrices of all the other links are fixed, thus only $\mathbf{Q}_l$ is adjusted. Therefore, for the sake of simplifying notations, we denote the sum data rate with $C(\mathbf{Q}_l)$. For link l to maximize the function $C(\mathbf{Q}_l)$, where $\mathbf{Q}_l \in \mathbf{\Phi}$ and $\mathbf{\Phi}$=$\{\mathbf{Q} \in C^{N_t \times N_t} | \mathbf{Q}$ is positive semidefinite, and $\mathrm{Tr}(\mathbf{Q}) \leq \bar{p}_l\}$ is a convex set, we start from an initial feasible point $\mathbf{Q}_l^{(1)} \in \mathbf{\Phi}$, then update the value according to

$$\mathbf{Q}_l^{(k+1)} = \mathbf{Q}_l^{(k)} + \alpha_k(\bar{\mathbf{Q}}_l^{(k)} - \mathbf{Q}_l^{(k)}) \tag{2.8}$$

where

$$\bar{\mathbf{Q}}_l^{(k)} = [\mathbf{Q}_l^{(k)} + s_k \nabla C(\mathbf{Q}_l^{(k)})]_{\mathbf{\Phi}} \tag{2.9}$$

Here, $0 < \alpha_k \leq 1$ is the stepsize, $s_k > 0$ is a scalar, $\nabla C(\mathbf{Q}_l^{(k)})$ is the gradient of

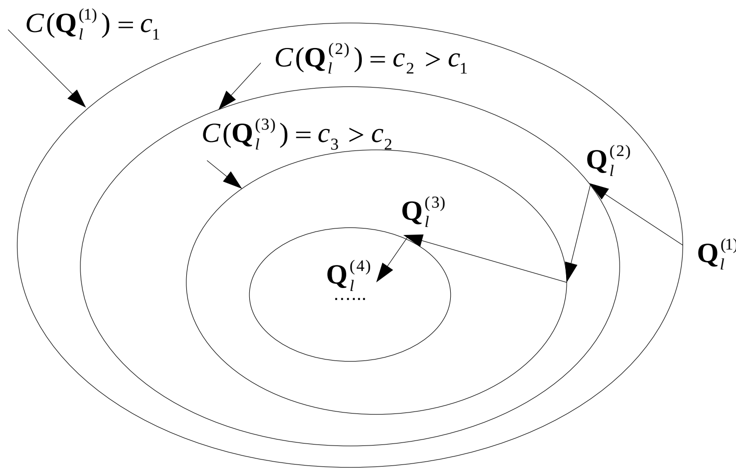

Figure 2.2: Iterative descent for maximizing a utility function [1].

$C(\mathbf{Q}_l)$ at the point $\mathbf{Q}_l^{(k)}$, and $[\cdot]_{\boldsymbol{\Phi}}$ denotes the projection on the convex set $\boldsymbol{\Phi}$. In this method, $\bar{\mathbf{Q}}_l^{(k)} - \mathbf{Q}_l^{(k)}$ is gradient related, which guarantees that there exists some α_k such that $C(\mathbf{Q}_l^{(k+1)}) > C(\mathbf{Q}_l^{(k)})$. $\mathbf{Q}_l^{(k)}$ is updated iteratively such that $C(\mathbf{Q}_l)$ is increased at each iteration (see Fig.2.2). According to the convergence analysis in [1], if α_k and s_k are chosen properly, the GP method always converges to a stationary point $\tilde{\mathbf{Q}}_l$, which satisfies $\mathrm{Tr}((\nabla C(\tilde{\mathbf{Q}}_l))^\dagger(\mathbf{Q}_l - \tilde{\mathbf{Q}}_l)) \leq 0$ for any $\mathbf{Q}_l \in \boldsymbol{\Phi}$. This method has a much higher computational complexity as it involves calculating gradients and projections, both on matrix variables. With this method, the transmitters are assumed to cooperate with a centralized control mechanism which has access to all of the channel state information and the covariance matrices of each user.

2.3 Game theoretic approach to power control

In the existing methods discussed above, each link uses the maximum power available to it, thus the power control is limited to spatial allocation among antenna elements. Although transmitting the highest power can result in high data rate, the energy efficiency might be low. In this section, we will propose a new technique for interference management in MIMO ad hoc networks using a game theoretic approach, in an effort to achieve both good sum data rate and energy efficiency.

2.3.1 Game Formulation

In the context of game theory, in adjusting its transmit power, each transmitter pursues a strategy that aims to maximize its utility. If we assume the cooperation between wireless links is not feasible, the problem can be modeled as a non-cooperative game (NCG), where each link is only concerned about its own utility rather than the system mutual information or system utility. Modeling this optimization process as an NCG, the three components that are present in the game are the following:

(1) Set of links: This is the set that contains all the links in the network. In the context of our network, we consider that all nodes are capable of both transmitting and receiving. However a node can act only as either a transmitter or receiver at a particular instance of time, thus there is no sharing of a transmitter or a receiver between links.

(2) Set of actions: The set of actions is basically the changes that a transmitter can make to help achieve equilibrium in the system. Since this is a non-cooperative game, global knowledge is not required. The transmitter changes the power allocations to increase the utility of its link in the face of changing interference. A practical constraint for a transmitter is that its transmit power cannot be higher than a certain value, i.e., $\mathrm{Tr}(\mathbf{Q}_l) \leq \bar{p}_l$.

(3) Set of utility functions: This set provides functional descriptions of individual preferences. Formally, a utility function is defined as follows [15].

Definition 1: A function that assigns a numerical value to the elements of the action set $u(\mathbf{A} \rightarrow \mathbb{R}^1)$ is a utility function, if for all $x, y \in \mathbf{A}$, x is at least as preferred compared to y if and only if $u(x) \geq u(y)$.

Putting together the three components shown above, the non-cooperative power control game can be expressed as: $G = [\mathbb{L}, \{\mathbb{A}_l\}, \{u_l(\cdot)\}]$, where $\mathbb{L}$ is the set of links, $\mathbb{A}_l$=$\{\mathbf{Q} \in C^{N_t \times N_t} | \mathbf{Q}$ is positive semidefinite, and $\mathrm{Tr}(\mathbf{Q}) \leq \bar{p}_l\}$ is the set of power allocation actions and $u_l(\cdot)$ is the utility function of link l. The transmit power of link l is limited as no higher than $\bar{p}_l$, while $\mathbb{A}_l$ is a convex set. We further denote the outcome of the game at certain time τ_k by the power allocation vector $\mathbf{q}(\tau_k) = (\mathbf{Q}_1(\tau_k), ..., \mathbf{Q}_L(\tau_k))$. In order to single out the action of link l, let $\mathbf{q}_{-l}(\tau_k)$ denote the vector consisting of elements of $\mathbf{q}(\tau_k)$ without the lth link. For any link l at time τ_k, the transmitter tries to find $\mathbf{Q}_l(\tau_k) \in \mathbb{A}_l$, such that for any other $\mathbf{Q}_l^{'}(\tau_k) \in \mathbb{A}_l$, $u_l(\mathbf{Q}_l(\tau_k), \mathbf{q}_{-l}(\tau_k)) \geq u_l(\mathbf{Q}_l^{'}(\tau_k), \mathbf{q}_{-l}(\tau_k))$. Each transmitter calculates its own link data rate and tries to optimize its utility function.

Nash equilibrium is a well-known solution to a non-cooperative game. It is defined as a strategy profile in which no player may gain from unilateral deviation from this profile [24]. For the game G, the definition of Nash equilibrium is as follows.

Definition 2: A power allocation vector $\mathbf{q}(\tau_k) = (\mathbf{Q}_1(\tau_k), ..., \mathbf{Q}_L(\tau_k))$ is defined to a Nash equilibrium point of $G = [\mathbb{L}, \{\mathbb{A}_l\}, \{u_l(\cdot)\}]$ if $u_l(\mathbf{Q}_l, \mathbf{q}_{-l}) \geq u_l(\mathbf{Q}_l^{'}, \mathbf{q}_{-l})$ holds for all $l \in \mathbb{L}$ and $\mathbf{Q}_l \in \mathbb{A}_l$.

Nash equilibrium does not necessarily exist, nor is it unique. It may be reached by each transmitter successively changing its power allocation in response to the interference environment from the other links in the network. All other links take the new allocation into consideration while calculating interference and the process is repeated for all links in the system until the power allocations for each user converges. The utility functions used most commonly in the application of game theory to communications are derived from SINR, BER [36, 30] and as in our case, link data rate.

2.3.2 Utility Function Design

Utility is the measure of "satisfaction" that a link obtains from using the channel, which is also called the payoff of the link. For a wireless ad hoc network, the utility function of a particular link l is related to the transmit power and the achievable data rate of that link, i.e., $u_l = u_l(p_l, C_l)$, where p_l and C_l are ultimately functions of $\mathbf{Q}_l$. We design a utility function which takes the following structure

$$u_l(p_l, C_l) = \frac{kg(C_l)}{p_l^{1/2}} \tag{2.10}$$

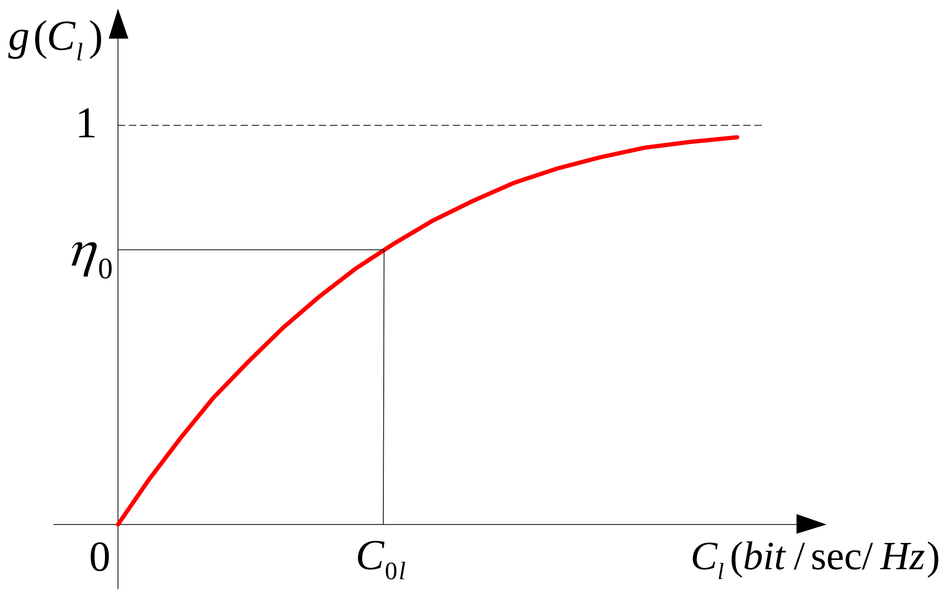

Figure 2.3: $g(C_l)$

where k is a positive constant whose unit is Volts so that the utility is a dimensionless number and $g(C_l) = 1 - (1 - \eta_0)^{C_l/C_{0l}}$ is plotted in Fig.2.3. Here C_{0l} is chosen to be a certain ratio (η_0) of the link's maximal data rate $C_{max,l}$, where $C_{max,l}$ is defined to be interference-free water-filling capacity of link l. It is obvious that if the transmit power for link l is given, maximum capacity occurs when none of the other links transmit and link l allocates power using water-filling. Equation(2.10) accommodates the mechanism of power control as the utility depends on both power and data rate. If interference is fixed and transmit power increases, link capacity and corresponding $g(C_l)$ does not change at the same rate as $p^{1/2}$. When p_l increases in the neighborhood of zero, $g(C_l)$ goes up faster than $p^{1/2}$ and u_l increases, thus the transmitter is encouraged to transmit more power so as to maximize utility. On the other hand, after p_l goes up to a certain level but still increases, $g(C_l)$ grows at a lower rate than $p^{1/2}$ and u_l goes down. Therefore, this discourages the transmitter from sending more power which would decrease utility. This intrinsic power control property is useful for a utility function in a power control game.

The utility function (2.10) is not perfect because it will result in a degenerate case in which maximum utility is achieved when all nodes transmit zero power. To avoid that degenerate situation, we modify Eq.(2.10) to be

$$u_l(\mathbf{Q}_l) = \frac{k}{\mathrm{Tr}(\mathbf{Q}_l)^{1/2}}(1 - 2(1-\eta_0)^{\frac{C_l}{C_{0l}}}) \tag{2.11}$$

From Eq.(2.10) to Eq.(2.11), we changed the coefficient of the exponential term to be 2. In addition, we switched the variables from (p_l, u_l) to $\mathbf{Q}_l$ because the former is determined by the latter, as shown by Eq.(2.2) and $p_l = \mathrm{Tr}(\mathbf{Q}_l)$.

The new utility function (2.11) has properties shown below.

Property 1: u_l is a monotonically increasing function of C_l for a fixed $p_l = \mathrm{Tr}(\mathbf{Q}_l)$. This can be shown by

$$\frac{\partial u_l}{\partial C_l} = \frac{k}{p_l^{1/2}}(-2\beta_l^{C_l}) \ln \beta_l > 0 \tag{2.12}$$

where $\beta = (1-\eta_0)^{1/C_{0l}} \in (0, 1)$.

Property 2: u_l obeys the law of diminishing marginal utility for large C_l given p_l is fixed, as shown by

$$\lim_{C_l \to \infty} \frac{\partial u_l}{\partial C_l} = \lim_{C_l \to \infty} \frac{k}{p_l^{1/2}}(-2\beta^{C_l}) \ln \beta_l = 0 \tag{2.13}$$

Property 3: u_l is monotonically decreasing when p_l increases, for a fixed C_l s.t. $C_l >$

$\log_\beta \frac{1}{2}$.

$$\frac{\partial u_l}{\partial p_l} = -\frac{k}{2p_l^{3/2}}(1 - 2\beta^{C_l}) < 0 \tag{2.14}$$

Property 4: u_l tends to zero in the limit that p_l goes to infinity.

$$\lim_{p_l \to \infty} u_l = \lim_{p_l \to \infty} \frac{k}{p_l^{1/2}}(1 - 2\beta^{C_l}) = 0 \tag{2.15}$$

For any link in an ad hoc network, power consumption and achieved data rate determine the payoff of that link. Properties 1 and 2 show that for a fixed transmit power in a link, the higher the data rate, the greater the payoff. However, the payoff tends to saturate as the data rate grows. In practical data transmission situations, link capacities are preferred to be high enough so as to maintain a certain degree of QoS. Once this requirement is met, however, the increment of satisfaction provided by additional link capacity will diminish until ultimately reaching saturation. Property 3 enforces that if the data rate of a link is high enough and can be guaranteed, transmitting more power is not desirable because it causes more interference to other links, thus bringing down the utility. The asymptotic behavior of transmitting a large amount of power is described in Property 4 which shows that extremely high transmit power results in zero satisfaction.

To further investigate the utility function for an ad hoc network, consider the two situations shown in Fig. 2.4. In both situations, node 1 transmitting to node 2 is the link of interest with the solid arrow showing desired communication links and the hatched arrow showing the propagation of interference. Fig. 2.4A illustrates the case that if the capacity of a particular link is more than enough to maintain a certain level of QoS, reducing the capacity by decreasing transmit power will mitigate the interference sent to other links. Since each link tries to maximize its utility, the transmitter will not be encouraged to transmit the maximum power. Another case of interest is shown in Fig. 2.4B. The achievable data rate for a particular link (node 1 to node 2) is low even when the transmitter sends data using maximum power. If such a low-rate but high power-consumption link exists, it has two major negative effects on the network. First, a low-rate link is not only useless in terms of data transmission but also it may bring down the data rate of other links due to generated interference. Second, the total power consumption of the network is inefficiently increased as other links may also transmit more power in order to counter interference. To avoid these negative effects, it is advisable to shut down low-efficiency links. The outcomes of utility-based power control with and without such a link shut-down mechanism will be shown in Section 2.4 of this chapter.

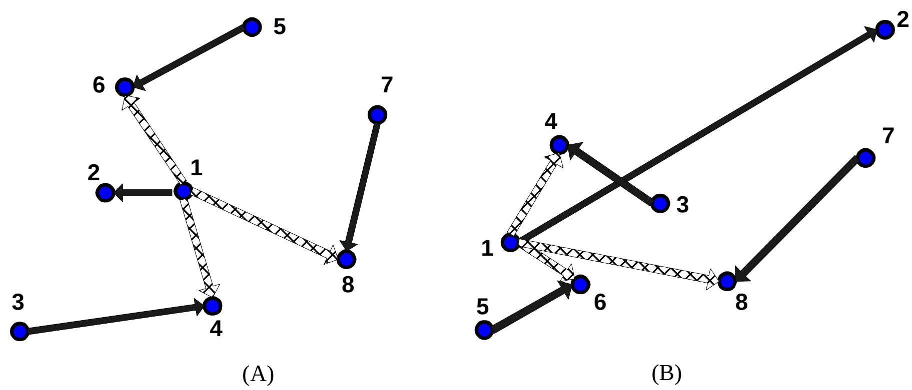

Figure 2.4: Illustration of the game theoretic approach with a mechanism for shutting down links (A) Situation in which link 1-2 should reduce transmit power (B) Situation in which link 1-2 should be shut off

Whether to shut down a particular link depends on the minimum data rate that is required by the network. For a non-priority network with homogeneous services, such as video-dedicated, each link has the same requirement for minimum data rate regardless of the topology of the network. Therefore, it is possible to assign a fixed threshold C_t which can be used to decide if a particular link should be shut down. This threshold is determined by the type of service that the network provides as well as the overall channel conditions which relate the QoS level to the threshold. Specifically, if the wireless channel conditions are good, the network administrator may set a high threshold so that all viable links will have high data rates because the minimum requirement is high. However, once a network is built up, C_t does not depend on the channel condition of each individual link due to the assumption that every link has the same QoS requirement for a homogeneous network.

Choosing the right (C_{0l}, η_0) is important in designing the utility function. To do so, we rewrite the condition in Property 3, i.e., $C_l > \log_\beta \frac{1}{2}$, by plugging $C_{0l} = \eta_0 C_{max,l}$ as

$$C_l > \log_\beta \frac{1}{2} = \eta_0 C_{max,l} \frac{\ln \frac{1}{2}}{\ln(1-\eta_0)} \tag{2.16}$$

C_l is the actual data rate of link l, which is greater than C_t due to the link shutting-down mechanism. Therefore, a more strict condition than (2.16) that guarantees Property 3 is

$$C_t > \eta_0 C_{max,l} \frac{\ln \frac{1}{2}}{\ln(1-\eta_0)} \tag{2.17}$$

which can be further converted to

$$\frac{\ln(1-\eta_0)}{\eta_0} < \frac{C_{max,l}}{C_t} \ln \frac{1}{2} \tag{2.18}$$

(2.18) shows how to choose η_0. It depends on the pre-determined threshold and the interference-free water-filling capacity of each link, where the latter is related to the network topology. In this chapter, we assume that each link in the network can find out the channel condition when other links are off, thus $C_{max,l}$ can be calculated for every l. (2.18) gives the value range for η_0. A different η_0 selected together with corresponding C_{0l} determines a unique utility function which leads to a different outcome of this power control game. In Section 2.4, simulation results will show how sensitive the system capacity and power consumption are to η_0. We also limit the network topology to a stationary case and assume that all nodes are powered on simultaneously. At start-up, each link calculates its multiuser water-filling capacity. If it is lower than the threshold, the link is shut down. Since the nodes are stationary, a low capacity link which has been shut down cannot be turned back on.

2.3.3 Game Theoretic Approach

Based on the above utility function, we propose an algorithm in which all links update their power allocation matrices iteratively. Initially all users agree on a certain capacity threshold C_t to keep the link viable and start with the same transmit power p_0 which is allotted equally to all antennas. Each link calculates its independent water-filling capacity [9] assuming there is no co-channel interference, and the multiuser water-filling capacity [39, 40] when interfering links are considered. If the multiuser capacity for link j is lower than the threshold C_t, then that link is shut down and the power allocation matrix $\mathbf{Q}_j$ is set to zero. For any viable link l, the transmitter calculates the optimum $p_l(0 \leq p_l \leq \bar{p}_l)$ such that the utility function is maximized. This p_l corresponds to a $\mathbf{Q}_l$ which is determined by maximizing (2.2). The power control process is performed iteratively until the utility for every link in the network converges.

Algorithm 1: Power control based on game theoretic approach

1 Initialization:

Set $k = 0$. Each link calculates its interference-free water-filling capacity and multiuser water-filling capacity, then decides on if the link should be shut down or not.

2 Update power allocations:

Let $k = k + 1$. For all links $l \in \mathbb{L}$, given power allocation vector $\mathbf{q}(\tau_{k-1})$, compute $\mathbf{Q}_l(\tau_k) = argmax_{\mathbf{Q}_l \in \mathbb{A}_l} = u_l(\mathbf{Q}_l, \mathbf{q}_{-l}(\tau_{k-1}))$. Repeat step 2 until the utility of every link converges.

2.3.4 Game Analysis

In the power control game $G = [\mathbb{L}, \{\mathbb{A}_l\}, \{u_l(\cdot)\}]$, the choice of power allocation matrix $\mathbf{Q}_l$ for user l impacts not only its own link capacity and utility, but also those of other links. Generally speaking, the link capacity in (2.2) and the utility in (2.11) are not concave or convex functions of $\mathbf{Q}_1, ..., \mathbf{Q}_L$. Therefore, analytical investigation of game properties without simplified assumptions is prohibitively difficult. In this section, we study the concavity of the utility function in the situation when interference for each link in the ad hoc network is extremely large, as one could assume in a dense outdoor network.

Theorem 1. *A Nash equilibrium exists in the NCG:* $G = [\mathbb{L}, \{\mathbb{A}_l\}, \{u_l(\cdot)\}]$ *when interference is sufficiently large for each link.*

Proof. For any link $l \in \mathbb{L}$, its data rate is determined by (2.2), which can be rewritten as

$$C_l = \log_2 \det(\mathbf{I} + \mathbf{Q}_l \mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1} \mathbf{H}_{l,l}) = \log_2 \det(\mathbf{I} + \mathbf{Q}_l \mathbf{U}_l \mathbf{\Sigma}_l \mathbf{U}_l^{\dagger}) \tag{2.19}$$

where the determinant identity $\det(\mathbf{I} + \mathbf{XY}) = \det(\mathbf{I} + \mathbf{YX})$ is used and $\mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1} \mathbf{H}_{l,l} = \mathbf{U}_l \mathbf{\Sigma}_l \mathbf{U}_l^{\dagger}$ is an eigenvalue decomposition of $\mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1} \mathbf{H}_{l,l}$, with $\mathbf{U}_l$ unitary and $\mathbf{\Sigma}_l = diag(\sigma_1, \sigma_2, ..., \sigma_{N_t})$.

When the transmitter sends out data with power p_l, the well-known water-filling algorithm can be used to obtain the optimum power allocation matrix $\mathbf{Q}_l$ that maximizes (2.19), which is given by

$$\mathbf{Q}_l = \mathbf{U}_l (\mu \mathbf{I} - \Sigma^{-1})^{+} \mathbf{U}_l^{\dagger} \tag{2.20}$$

where μ is known as the water level and is chosen to satisfy $\mathrm{Tr}(\mu \mathbf{I} - \mathbf{\Sigma}^{-1})^{+} = p_l$. $(\mu \mathbf{I} - \mathbf{\Sigma}^{-1})^{+}$ is a diagonal matrix with rank r where $r \leq N_t$ and we denote it by $\mathbf{\Omega}_l = diag\{\omega_1, \omega_2, ..., \omega_r, 0, ..., 0\}$. Substituting $\mathbf{\Omega}_l$ into (2.20) and (2.19) yields the capacity of link l as

$$\begin{aligned} C_l &= \log_2 \det(\mathbf{I} + \mathbf{U}_l \mathbf{\Omega}_l \mathbf{U}_l^{\dagger} \mathbf{U}_l \mathbf{\Sigma}_l \mathbf{U}_l^{\dagger}) \\ &= \log_2 \det(\mathbf{I} + \mathbf{\Omega}_l \mathbf{\Sigma}_l) \\ &= \sum_{i=1}^{r} \log_2 (1 + \omega_i \sigma_i) \end{aligned} \tag{2.21}$$

Given interference to link l is sufficiently large, the eigenvalues of $\mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1} \mathbf{H}_{l,l}$ are diminishingly small. We further assume all these eigenvalues are close to each other, i.e.,

$\sigma_i \approx \sigma_0$. Therefore, (2.21) can be approximated as

$$C_l \approx \frac{1}{\ln 2}\sum_{i=1}^{r}\omega_i\sigma_i \approx \frac{\sigma_0}{\ln 2}\sum_{i=1}^{r}\omega_i = \frac{\sigma_0}{\ln 2}p_l \tag{2.22}$$

Plugging the above approximation into (2.11), we obtain a new utility function as

$$u_l(p_l) = \frac{k}{p_l^{1/2}}(1 - 2\beta^{\frac{\sigma_0}{\ln 2}p_l}) = \frac{k}{p_l^{1/2}}(1 - 2\alpha^{p_l}) \tag{2.23}$$

where $\alpha = \beta^{\frac{\sigma_0}{\ln 2}}$ is close to 1 because of sufficiently small σ_0.

Given interference seen by link l, the power allocation matrix $\mathbf{Q}_l$ is uniquely determined by p_l[40], thus the power control game G can be equivalently stated as NCG: $S = [\mathbb{L}, \{\mathbb{P}_l\}, \{u_l(\cdot)\}]$, where $\mathbb{P}_l$ is the strategy set in terms of transmit power and we denote the outcome of the game by the transmit power vector $\mathbf{p} = (p_1, p_2, ..., p_L)$. In [29, 15], it has been shown that a Nash equilibrium exists, if for any l:

(1) $\mathbb{P}_l$ is a nonempty, convex and compact subset of some Euclidean space $\mathbb{R}^L$.

(2) $u_l(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_l.

Each link has a strategy space that is given by $0 \le p_l \le \bar{p}_l$. For any $p_l^{(1)}, p_l^{(2)} \in \mathbb{P}_l$ and $0 < \gamma < 1$, $p_1^{(3)} = \gamma p_l^{(1)} + (1-\gamma)p_l^{(2)} \in \mathbb{P}_l$. Thus the first condition is satisfied. Eq.(2.23) indicates that u_l is a continuous function in $\mathbf{p}$. Next we will show that Eq.(2.23) is concave in p_l. We approach this problem by investigating the second-order derivation of u_l with respect to p_l.

$$\begin{aligned}\frac{\partial^2 u_l}{\partial p_l^2} &= kp_l^{-\frac{5}{2}}[0.75(1-2\alpha^{p_l}) + 2\alpha^{p_l}(1 - p_l\ln\alpha)p_l\ln\alpha] \\ &\le kp_l^{-\frac{5}{2}}[0.75(1-2\alpha^{p_l}) + 0.5\alpha^{p_l}] \\ &= kp_l^{-\frac{5}{2}}(0.75 - \alpha^{p_l}) < 0 \end{aligned} \tag{2.24}$$

where in the first inequality $x(1-x) \le 1/4$ ($x \in \mathbb{R}$) is used and in the second one α^{p_l} is approximated to 1 since α is close to 1.

(2.24) indicates that u_l is a concave function in p_l. Because both conditions are satisfied, there exists a Nash equilibrium for the power control game $G = [\mathbb{L}, \{\mathbb{A}_l\}, \{u_l(\cdot)\}]$ when interference is sufficiently large[1]. □

Theorem 2. *The NCG: $S = [\mathbb{L}, \{\mathbb{P}_l\}, \{u_l(\cdot)\}]$ has a unique equilibrium point when interference is sufficiently large for each link.*

Proof. Theorem 4.4 in [12] shows that under certain conditions a game has at most one equilibrium point. In the context of our power control game, those conditions are formulated as

(i) $\mathbb{P}_l = \{p_l \in \mathbb{R}, g_l(p_l) = \bar{p}_l - p_l \geq 0\}$ is nonempty and g_l is a continuously differentiable concave function in an open set containing P_l for each $l = 1, ..., L$.

(ii) There exists a $\tilde{p}_l \in P_l$ to satisfy $g_l(\tilde{p}_l) > 0$ for all $l = 1, ..., L$.

(iii) All payoff functions u_l are concave in p_l with fixed values of p_j $(j \neq l)$ and twice continuously differentiable in an open set containing $P = P_1 \times ... \times P_L$.

(iv) Game S is diagonally strictly concave on P, i.e., for any $\mathbf{p}^{(0)} \neq \mathbf{p}^{(1)}$, $\mathbf{p}^{(0)}, \mathbf{p}^{(1)} \in P$ and for some $\mathbf{t} \geq 0$ $(\mathbf{t} \in \mathbb{R}^L)$, the following inequality holds.

$$(\mathbf{p}^{(1)} - \mathbf{p}^{(0)})\mathbf{h}(\mathbf{p}^{(0)}, \mathbf{t}) + (\mathbf{p}^{(0)} - \mathbf{p}^{(1)})\mathbf{h}(\mathbf{p}^{(1)}, \mathbf{t}) > 0 \tag{2.25}$$

where the function $\mathbf{h} : \mathbb{R}^L \longrightarrow \mathbb{R}^L$ is defined as

$$\mathbf{h}(\mathbf{p}, \mathbf{t}) = \begin{bmatrix} t_1 \frac{\partial u_1}{\partial p_1} \\ ... \\ t_L \frac{\partial u_L}{\partial p_L} \end{bmatrix} \tag{2.26}$$

Next we will show that the game $S = [\mathbb{L}, \{P_l\}, \{u_l(\cdot)\}]$ satisfies all of the conditions above. For (i) and (ii), $g_l(p_l) \geq 0$ comes from the power constraint and $g_l(p_l)$ is a linear function with respect to p_l, thus (i) and (ii) are readily fulfilled. It has been proven in Theorem 1 that the utility function u_l is a concave function of p_l in case of sufficiently large interference, thus (iii) is satisfied. The condition (iv) requires that (2.25) hold. To

[1]As shown in the proof, 'sufficiently large' means $\sigma_0 \approx \sigma_1 \approx \ldots \approx \sigma_r$ and $\log_2(1 + \omega_i\sigma_i) \approx \omega_i\sigma_0/\ln 2$

show that, we plug (2.26) into (2.25) and rewrite the LHS in an element-wise format as the following

$$\begin{aligned} LHS &= (\mathbf{p}^{(1)} - \mathbf{p}^{(0)})[\mathbf{h}(\mathbf{p}^{(0)}, \mathbf{t}) - \mathbf{h}(\mathbf{p}^{(1)}, \mathbf{t})] \\ &= (p_1^{(1)} - p_1^{(0)}, ..., p_L^{(1)} - p_L^{(0)}) \begin{bmatrix} (\frac{\partial u_1}{\partial p_1^{(0)}} - \frac{\partial u_1}{\partial p_1^{(1)}})t_1 \\ ... \\ (\frac{\partial u_L}{\partial p_L^{(0)}} - \frac{\partial u_L}{\partial p_L^{(1)}})t_L \end{bmatrix} \\ &= \sum_{l=1}^{L} t_l (p_l^{(1)} - p_l^{(0)}) \Big(\frac{\partial u_l}{\partial p_l^{(0)}} - \frac{\partial u_l}{\partial p_l^{(1)}} \Big) \end{aligned} \quad (2.27)$$

where transmit power vector $\mathbf{p}^{(i)} = (p_1^{(i)}, ..., p_L^{(i)})$ for $i = 0, 1$ and $\mathbf{t} = (t_1, ..., t_L) \geq \mathbf{0}$.

Given $\mathbf{p}^{(0)} \neq \mathbf{p}^{(1)}$, $\exists j \in \mathbb{L}$, s.t. $p_j^{(0)} \neq p_j^{(1)}$. If $p_j^{(0)} < p_j^{(1)}$, because of the concavity of $u_j(p_j)$, $\partial u_j / \partial p_j$ is monotonically decreasing on p_j, which yields $\partial u_j / \partial p_j^{(0)} > \partial u_j / \partial p_j^{(1)}$. Therefore, we have

$$t_j (p_j^{(1)} - p_j^{(0)}) \Big(\frac{\partial u_j}{\partial p_j^{(0)}} - \frac{\partial u_j}{\partial p_j^{(1)}} \Big) > 0 \quad (2.28)$$

and (2.27) is positive. If $p_j^{(0)} > p_j^{(1)}$, similarly we can show (2.27) is positive. Therefore, condition (iv) is satisfied. Since all of the requirements in (i)-(iv) are met, the game S where interference is sufficiently large has at most one equilibrium point. According to Theorem 1, there exists a Nash equilibrium point in the game S, therefore it is unique. $\square$

Note that the concavity of the utility function is a sufficient condition for the uniqueness of Nash equilibrium, but it is not a necessary one. Thus, while the numerical simulations in the next section may not meet the requirement for sufficiently large interference for each link, a unique Nash equilibrium is found for our simulations.

2.4 Simulation Results

The ad hoc network is simulated in downtown Philadelphia (shown in Fig. 2.5) with computational electromagnetics [10]. The topology is static and contains transmit-receive nodes 5-15 (link 1), 8-11 (link 2), 14-3 (link 3), 10-2 (link 4), 1-6 (link 5) and 4-9 (link 6). All of these links are single-hop and no node relays information. We compare the sum data rate and energy efficiency under different methods, namely, game theoretic approach with link shut-down mechanism (GTWS) or without that mechanism (GTWOS), gradient projection (GP) approach and multiuser water-filling (MUWF) approach. Since power control is applied to the game theoretic approach, each individual link may not transmit the same amount of power even though the initial transmit power of each link is the same. In order to compare the performance of different methods in a fair way, we fix the total power consumption of the network, which is determined by the GTWS technique, and divide power among links. Specifically, we first set each link to transmit the same amount of power and use GTWS to compute the sum data rate and total power consumption($\sum_{l=1}^{6} p_l$), then divide the total power($\sum_{l=1}^{6} p_l$) equally to all 6 links and compute sum data rate using MUWF and GP method. When applying the GTWOS method, we let each link start with equal transmit power ($\frac{1}{6}\sum_{l=1}^{6} p_l$) but assign a different maximum power constraint in order to get the same total power consumption as GTWS and thus make a fair comparison.

Fig. 2.6 shows the sum data rate of the network for different algorithms. The SNR is calculated on the basis of equal transmit power for GP and MUWF methods. We can see that for every SNR the GTWS method achieves the highest sum data rate while the MUWF method results in the lowest system capacity[2]. The GP method and MUWF method assume that each transmitter sends a constant amount of power no matter how inefficient that particular link is, which might cause severe interference to other links. The same situation could happen in the GTWOS method as well, where inefficient links may

[2]The convergence point of the GP method depends on the initial condition. However, [37] found out the ergodic mutual information curves are extremely close to each other and one choice of initial condition is not evidently better than another. In this dissertation, the initial condition is set to equal power allocation.

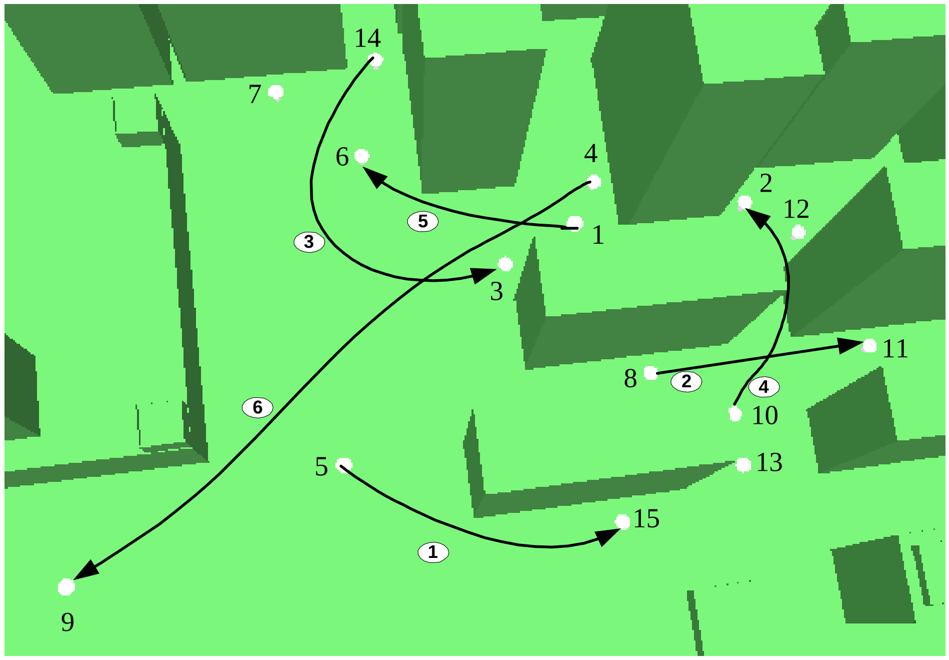

Figure 2.5: Illustration of Philadelphia downtown simulation. Node numbers are not circled and link numbers are circled.

transmit even higher power because power control is allowed. For our proposed method, inefficient links are shut down so as to avoid a waste of power. For instance, in the simulation at SNR = 19.6dB, link 1 has such a low capacity that it is unusable. As a result, it is shut off. However, the sum capacity is still higher than the other methods, which indicates that although shutting off deficient links reduces the number of users in the network, it improves the data rate of existing users.

Among the three approaches without the mechanism of link shut-down, the GP method results in the highest sum data rate. This is reasonable because its objective function is to maximize the sum data rate while in the other two methods each link aims to maximize its own data rate or utility. GTWOS and MUWF are both game-based algorithms[40][37], in which the selfish utility function introduces conflicts among links and power allocations at the Nash equilibrium are less efficient than possible power allocations acquired through cooperation. Comparing GTWOS and MUWF methods, GTWOS leads to a higher sum data rate and energy efficiency. This is because a power control mechanism has been implicitly built into the utility function (2.11) where a transmitter is not encouraged to transmit high power in order to obtain high capacity, but to maximize its utility.

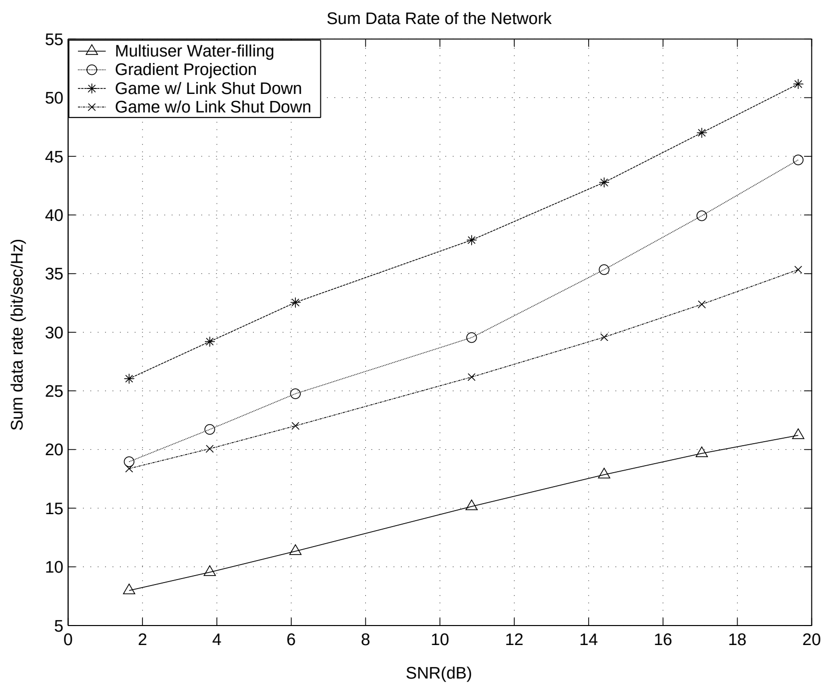

Figure 2.6: Sum data rate of different methods

Fig. 2.7 demonstrates the energy efficiency for different methods. A higher value of λ means a higher capacity achieved per unit energy. We can see that for every SNR the game theoretic approach has the highest λ and thus the energy utilization is the most efficient. Shutting off a deficient link not only saves the battery life of the node in a particular link, but also reduces interference to other links, allowing other nodes to transmit with less power.

When the GTWS method is used, Fig.2.8 shows how sensitive the system capacity and total power consumption are to different values of η_0 (as given by (2.11)). It can be

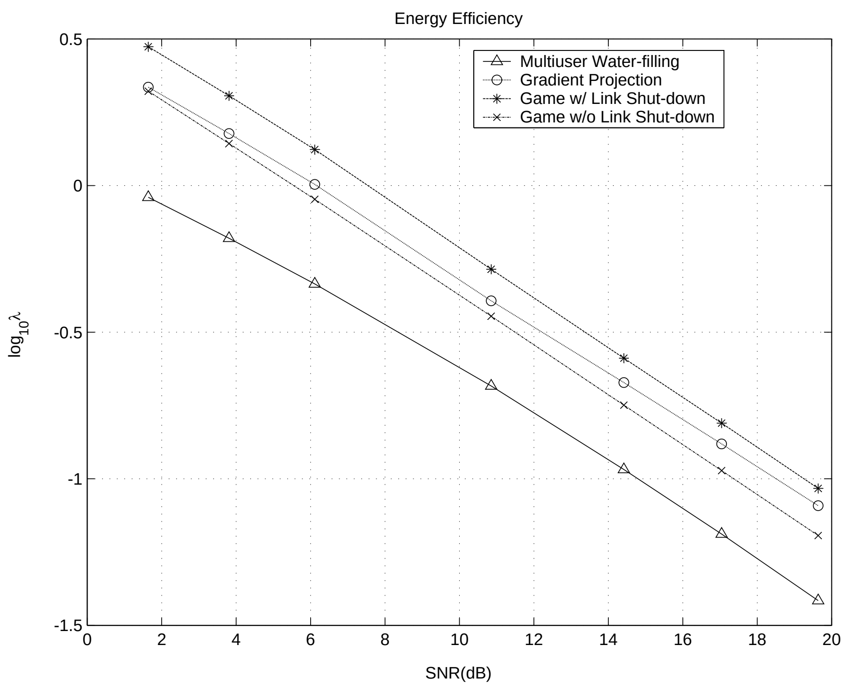

Figure 2.7: Energy efficiency of different methods

seen that a higher η_0 tends to result in a lower total transmit power and consequently a lower sum data rate. However, with increasing η_0, it is easier to satisfy condition (2.16). Therefore, there would be a trade-off when selecting η_0: If η_0 is chosen to be a small value, the system capacity will be large, but (2.16) may not hold. It is suggested to select the smallest η_0 that satisfies (2.16) while maximizing the system capacity.

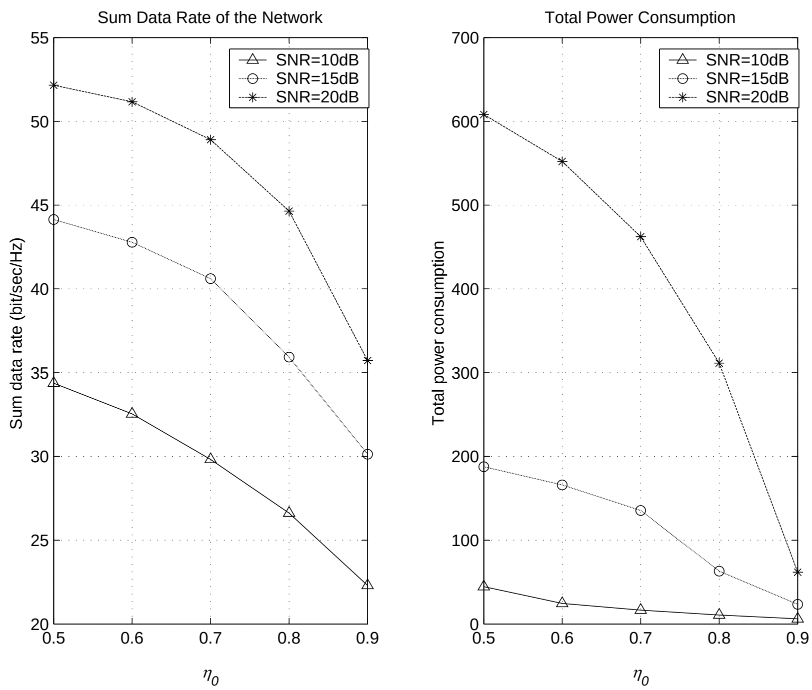

Figure 2.8: System capacity and power consumption vs η_0

2.5 Summary

In this chapter, the topic of power management in MIMO ad hoc networks was addressed. Existing approaches such as multiuser water-filling and gradient projection assign a fixed transmit power to each link and each transmitter node allocates power among different antennas in order to optimize the link capacity or sum data rate. If bad channel conditions existed in some communicating links, those methods are not energy efficient.

We proposed a new technique for power management and interference reduction based upon a game theoretic approach. A utility function with an intrinsic property of power control was designed and power allocation in each link was built into a non-cooperative game. To avoid unnecessary power transmission under poor channel conditions, a mechanism of shutting down inefficient links was integrated into the game theoretic approach. Simulation results showed that under the constraint of a fixed total of transmit power, if the proposed approach were allowed to shut off deficient links, the remaining links would still achieve the highest sum data rate and energy efficiency.

We also investigated how to select key parameters for the utility function. Based on the assumption that the non-priority network contained homogenous service and that all nodes were stationary, a fixed capacity threshold C_t was assigned to the network and accordingly the condition for the ratio to maximum data rate η_0 was found. It was shown that the sum data rate and total transmit power were sensitive to η_0 and it was also shown that there was a trade-off between data rate and utility function requirement when selecting η_0.

3. Power Management in MIMO Ad Hoc Networks - A Soft Shut-down Mechanism

In Chapter 2, a game theoretic approach to power management in a MIMO ad hoc network was presented. The power allocation in each link is built into a non-cooperative game where a utility function is identified and maximized. In order to reduce co-channel interference and improve energy efficiency, a mechanism for shutting down links is proposed. In Algorithm 2.3.3, whether to shut down a link is determined by its data rate from multiuser water-filling method. If it is lower than a pre-set threshold, the link is shut down with its power allocation matrix set to zero. This is a hard shut-down mechanism, because the link is terminated by being forced to send no power. The game analysis based on this hard mechanism is only applicable to those viable links. In real situations, the channel condition of a link may change from time to time and the interference that link experiences also fluctuates. For instance, interfering transmitters may move towards or away from the receive node of the link of interest, or interfering links may drop out of the network, etc. All those factors affect the data rate of a link of interest, so it will be beneficial to set up a mechanism which allows a link to adaptively control its transmit power. This mechanism is expected to turn off a link if its data rate is too low, but allows a link to transmit if channel condition has improved, therefore it is called a soft shut-down mechanism. In this chapter, we will carefully design a new utility function in order to accommodate the mechanism described.

3.1 Power Control Game with a Soft Shut-down Mechanism

In this section, we will design a new utility function which allows a link to be shut down or turned back on by tuning a pricing factor. How to choose the pricing factor will be discussed thoroughly and a game analysis will be provided in the end.

3.1.1 Utility Function Design

For a wireless ad hoc network, we relate the utility function of a particular link l to the achievable data rate of that link, i.e., $g_l = C_l$. This relationship quantifies approximately the demand or willingness of the user to pay for a certain level of service. (2.2) gives the instantaneous data rate of a MIMO link in the presence of interference. The expression involves matrix variables $(\mathbf{Q}_1, \ldots, \mathbf{Q}_L)$, which can be simplified to vector variables as shown by Lemma 3.

Lemma 3. *Given all other links' power allocation matrices $\mathbf{Q}_j (j \neq l)$, maximizing link l's data rate with respect to $\mathbf{Q}_l$ can be formulated as the following constrained optimization problem*

$$\begin{aligned} \max_{\mathbf{z}_l} \quad & \sum_{i=1}^{N_t} \log_2(1 + z_{l,i}\sigma_{l,i}) \\ s.t. \quad & \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l \\ & z_{l,i} \geq 0, \quad 1 \leq i \leq N_t \end{aligned} \tag{3.1}$$

where $\sigma_{l,i}$ are the eigenvalues of $\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l}$ and $\mathbf{z}_l = (z_{l,1}, ..., z_{l,N_t}) \in \mathbb{R}_+^{N_t}$ are the eigenvalues of $\mathbf{Q}_l$ with $z_{l,i}$ representing the power for the i^{th} eigenmode.

Proof. Using eigenvalue decomposition $\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l} = \mathbf{U}\mathbf{\Sigma}_l\mathbf{U}^{\dagger}$, where $\mathbf{\Sigma}_l = diag\{\sigma_{l,1}, \ldots, \sigma_{l,N_t}\}$ and $\mathbf{U}$ is unitary, we have

$$C_l = \log_2 \det\left(\mathbf{I} + \mathbf{Q}_l\mathbf{U}\mathbf{\Sigma}_l\mathbf{U}^{\dagger}\right) = \log_2 \det\left(\mathbf{I} + \mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U}\mathbf{\Sigma}_l\right) \tag{3.2}$$

Denoting $\mathbf{A} = \mathbf{I} + \mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U}\mathbf{\Sigma}_l$, we have

$$\mathbf{A}\mathbf{A}^{\dagger} = \mathbf{I} + \mathbf{\Sigma}_l\mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U} + \mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U}\mathbf{\Sigma}_l + \mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U}\mathbf{\Sigma}_l^2\mathbf{U}^{\dagger}\mathbf{Q}_l\mathbf{U} \tag{3.3}$$

According to Hadamard's inequality [23],

$$|\det(\mathbf{A})|^2 \leq \prod_{i=1}^{N_t} \left(\sum_{j=1}^{N_t} |a_{ij}|^2 \right) \tag{3.4}$$

with equality if and only if $\mathbf{A}\mathbf{A}^\dagger$ is diagonal. Therefore, in order to maximize (3.2), (3.3) should be diagonal. Since $\mathbf{I}$ and $\mathbf{\Sigma}_l$ are already diagonal matrices, we only need to set $\mathbf{U}^\dagger \mathbf{Q}_l \mathbf{U}$ to be diagonal. To do so, let $\mathbf{U}^\dagger \mathbf{Q}_l \mathbf{U} = diag\,(z_{l,1}, \ldots, z_{l,N_t})$ and (3.2) is converted to

$$C_l = \log_2 \prod_{i=1}^{N_t} (1 + z_{l,i}\sigma_{l,i}) = \sum_{i=1}^{N_t} \log_2(1 + z_{l,i}\sigma_{l,i}) \tag{3.5}$$

with the constraint $\mathrm{Tr}(\mathbf{Q}_l) = \mathrm{Tr}(\mathbf{U}^\dagger \mathbf{Q}_l \mathbf{U}) = \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l$ and $z_{l,i} \geq 0, \forall i$. □

In an ad hoc network, each link maximizes its own data rate at the cost of high power consumption, which also causes interference to other users and brings down their data rates. In order to keep a link from *selfishly* transmitting the highest power available, the system should impose a pricing function. For the sake of simplicity, our pricing function is proportional to link transmit power. As a result, the net utility of the l^{th} link is given by

$$v_l = C_l - \gamma_l p_l \tag{3.6}$$

where γ_l is a nonnegative scaling factor whose units are bps/Hz/Watt so that the two terms in (3.6) have the same units. In this paper, we assume there exists an external network controller which assigns a value to γ_l. Different from the central control in the GP method, this external controller does not pass information of channel state and power allocation matrices. Instead, it provides a consensus among all links as to how γ_l is set.

Since each link tries to maximize its net utility which contains transmit power as the pricing term, the transmitter will not be encouraged to transmit at maximum power. In this situation, the pricing factor γ_l enforces the minimum required capacity per unit power. One possible formulation of the objective function in (3.6) has

$$\gamma_l = \frac{\alpha_l}{p_0} \tag{3.7}$$

where α_l is a certain capacity value and p_0 is the initial transmit power. This equation describes the pricing factor for the General Game-Theoretic (GGT) technique that will be further described in Section 3.3.

In this chapter, we will also investigate a similar link shut-down mechanism as discussed in Chapter 2. This mechanism can be implemented by setting γ_l to be a large value if a threshold capacity is not exceeded. In this formulation,

$$\gamma_l = \begin{cases} \frac{\alpha_l}{p_0} & C_{l,0} \geq C_l^t \\ \infty & C_{l,0} < C_l^t \end{cases} \tag{3.8}$$

where $C_{l,0}$ is the initial multiuser water-filling capacity of link l [40, 39] and C_l^t is a capacity threshold assigned to link l by the external network controller. This equation describes the pricing factor for the Game-Theoretic technique with link Shutdown (GTWS) that will be further described in Section 3.1.4.

With the new net utility as the objective function, each link tries to optimize (3.6). This problem can be formulated as:

$$\begin{aligned} \max_{\mathbf{z}_l} \quad & \sum_{i=1}^{N_t} \log_2(1 + z_{l,i}\sigma_{l,i}) - \gamma_l \sum_{i=1}^{N_t} z_{l,i} \\ s.t. \quad & \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l \\ & z_{l,i} \geq 0, \quad 1 \leq i \leq N_t \end{aligned} \tag{3.9}$$

3.1.2 Game Formulation

In the context of game theory, if cooperation between wireless links is assumed to be infeasible, the problem can be modelled as a non-cooperative game(NCG): $T = [\mathbb{L}, \{\mathbb{B}_l\}, \{v_l(\cdot)\}]$, where $\mathbb{L}$ is the set of links, $\mathbb{B}_l$=$\{\mathbf{z}_l \in \mathbb{R}_+^{N_t} | \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l\}$ is the set of power allocation actions and $v_l(\cdot)$ is the utility function of link l. We further denote the outcome of the game at certain time τ_k by the power allocation vector $\mathbf{q}(\tau_k) = (\mathbf{z}_1(\tau_k), ..., \mathbf{z}_L(\tau_k))$. In order to single out the action of link l, let $\mathbf{q}_{-l}(\tau_k)$ denote the vector consisting of elements of $\mathbf{q}(\tau_k)$ without the l^{th} link. For any link l at time τ_k, the transmitter tries to find $\mathbf{z}_l(\tau_k) \in \mathbb{B}_l$, such that for any other $\mathbf{z}_l^{'}(\tau_k) \in \mathbb{B}_l$, $v_l(\mathbf{z}_l(\tau_k), \mathbf{q}_{-l}(\tau_k)) \geq v_l(\mathbf{z}_l^{'}(\tau_k), \mathbf{q}_{-l}(\tau_k))$. Each transmitter calculates its own link capacity and tries to optimize its utility function.

3.1.3 Iterative Power Control with Game Theory

Based on the above utility function, we propose an algorithm in which all links update their power allocation vectors iteratively. We assume that all nodes are powered on simultaneously. Initially each link is given a certain capacity threshold C_l^t to keep the link viable and starts with a fixed transmit power p_0 which is allotted equally to all antennas. For each link its multiuser water-filling capacity $C_{l,0}$ is calculated and a corresponding γ_l is assigned based upon Equation (3.7) or Equation (3.8). For any viable link l, the transmitter calculates the optimum $p_l (0 \leq p_l \leq \bar{p}_l)$ such that the net utility function is maximized. This p_l corresponds to a $\mathbf{z}_l$ which is determined by maximizing (3.9). The power control process is performed iteratively until the net utility for every link in the network converges.

Algorithm: Power control based on game theoretic approach

1 Initialization:

Set $k = 0$. Each link calculates its multiuser water-filling capacity, then a γ_l is assigned based upon Equation (3.7)(GGT method) or Equation (3.8) (GTWS method).

2 Update power allocations:

Let $k = k+1$. For all links $l \in \mathcal{L}$, given power allocation vector $\mathbf{q}(\tau_{k-1})$, compute $\mathbf{z}_l(\tau_k) = \arg\max_{\mathbf{z}_l \in \mathbb{B}_l} u_l(\mathbf{z}_l, \mathbf{q}_{-l}(\tau_{k-1}))$. Repeat step 2 until the net utility of every link converges.

3.1.4 Implication of Pricing Factor γ_l on Power Allocation

We now show how to choose the right parameter γ_l for the net utility function so as to accommodate the link shutdown mechanism and to improve power efficiency of the network. To do so, we approach the optimization problem (3.9) with Lagrange multiplier theory. The Lagrangian function in a standard minimization formulation can be written as

$$L(\mathbf{z}_l, \mathbf{u}) = \sum_{i=1}^{N_t} \left[\gamma_l z_{l,i} - \log_2(1 + z_{l,i}\sigma_{l,i})\right] + \mu_0 \left(\sum_{i=1}^{N_t} z_{l,i} - \bar{p}_l\right) - \sum_{i=1}^{N_t} \mu_i z_{l,i} \qquad (3.10)$$

where $\mathbf{u} = (\mu_0, ..., \mu_{N_t})$ is the Lagrange multiplier vector and $\sigma_{l,1}, ..., \sigma_{l,N_t}$ are constants due to the assumption that all the other links' power allocations ($\mathbf{q}_{-l}$) are given.

According to Karush-Kuhn-Tucker(KKT) necessary conditions [1], let $\mathbf{z}_l$ be a local maximum of (3.9), then there exists a unique $\mathbf{u}$, such that

$$\frac{\partial L}{\partial z_{l,i}} = \gamma_l - \frac{\sigma_{l,i}}{(1 + \sigma_{l,i} z_{l,i}) \ln 2} + \mu_0 - \mu_i = 0, \quad i = 1, ..., N_t \qquad (3.11)$$

$$\mu_j \geq 0, \quad j = 0, 1, ..., N_t \qquad (3.12)$$

This optimization problem contains all inequality constraints. For any feasible point $\mathbf{z}_l$, if inequality strictly holds for a certain constraint $g_j(\mathbf{z}_l) \leq 0$, the corresponding Lagrange multiplier $\mu_j = 0$ and this constraint is called inactive. Conversely, if $\mu_j > 0$, then that constraint is active because $g_j(\mathbf{z}_l) = 0$.

Theorem 4. *Link l will be shut down, if $\gamma_l > \sigma_{l,max}/\ln 2$, where $\sigma_{l,max} = \max_i(\sigma_{l,i})$.*

Proof. The KKT condition (3.11) can be converted to

$$\frac{1}{\gamma_l + \mu_0 - \mu_i} = \left(z_{l,i} + \frac{1}{\sigma_{l,i}}\right) \ln 2 \qquad (3.13)$$

Given $\gamma_l > \sigma_{l,max}/\ln 2$ and from (3.12), $\mu_0 \geq 0$, so we know

$$\frac{1}{\gamma_l + \mu_0} \leq \frac{1}{\gamma_l} < \frac{\ln 2}{\sigma_{l,i}} \leq \left(z_{l,i} + \frac{1}{\sigma_{l,i}}\right) \ln 2 \tag{3.14}$$

Comparing (3.13) and (3.14), we see that (3.13) is true only if $\mu_i > 0$, which indicates the constraint $z_{l,i} \geq 0$ is active for any $i = 1, ..., N_t$. Since all $z_{l,i} = 0$, the transmit power of link l is zero and it is shut down. □

Theorem 4 shows that if γ_l is large enough, link l does not transmit power. In real situations, a transmit node acquires channel information about the link of interest as well as the interfering links, and calculates its data rate and $\sigma_{l,max}$. If the initial multiuser water-filling capacity is lower than the preset threshold, the link is shut down. This mechanism can be implemented by assigning a sufficiently large value to γ_l.

Another strategy for choosing γ_l is to select it to be very small. In the extreme situation when $\gamma_l = 0$, which means no pricing is imposed in the utility function, the power allocation result is obviously the well-known water-filling solution with maximum power transmitted. Generally, when γ_l is smaller than a certain value ζ_l, link l uses the maximum power available to it and the power allocation is given by $z_{l,i} = \left(\frac{1}{(\gamma_l+\mu_0)\ln 2} - \frac{1}{\sigma_{l,i}}\right)^+$, where μ_0 is chosen to satisfy $\sum_{i=1}^{N_t} z_{l,i} = \bar{p}_l$. ζ_l is defined here as the cut-off value for power allocation strategies using maximum power versus strategies that use less transmit power. It is nontrivial to give an analytical expression for ζ_l because of the complex relationship between $\sigma_{l,i}$ and $\bar{p}_l$.

Fig.3.1 illustrates different strategies for power allocation in link l with respect to different values of γ_l. The region of $\zeta_l < \gamma_l < \frac{1}{\ln 2}\sigma_{l,max}$ corresponds to a transmit power p_l which is between 0 and $\bar{p}_l$. Mathematically, the constraint on link l's maximum transmit power is inactive, so this link only uses a portion of the power available to it. This situation is desirable for higher energy efficiency, as long as this reduced power consumption allows QoS requirements to be met. In order to avoid unnecessarily transmitting at the highest power which potentially harms network energy efficiency, link l can choose to increase the value of γ_l. In Section 3.3, simulation results will show how γ_l affects the link data rate and power consumption.

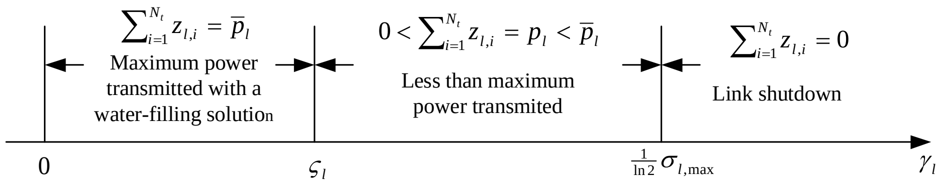

Figure 3.1: Power allocation vs γ_l

In the discussion above, it was assumed that the interference to link l is fixed because all the other links' power allocations do not change ($\sigma_{l,1}, ..., \sigma_{l,N_t}$ are constants). However, during the course of power control, each link adjusts its power allocation from iteration to iteration until power allocation converges for every link ($\sigma_{l,1}, ..., \sigma_{l,N_t}$ fluctuate). For example, suppose at time τ_{k+1} every link except l increases its transmit power to θ times that of at time τ_k, i.e., $\mathbf{q}_{-l}(\tau_{k+1}) = \theta \mathbf{q}_{-l}(\tau_k)$ with $\theta > 1$, and assume $\mathbf{R}_l(\tau_k) \gg \mathbf{I}$ element-wise, then it can be shown that $\mathbf{R}_l(\tau_{k+1}) = \mathbf{I} + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{Q}_j(\tau_{k+1})\mathbf{H}_{l,j}^{\dagger} \approx \theta \mathbf{R}_l(\tau_k)$ and $\sigma_{l,max}(\tau_{k+1}) \approx \frac{1}{\theta}\sigma_{l,max}(\tau_k) < \sigma_{l,max}(\tau_k)$. Similarly, $\sigma_{l,max}(\tau_{k+1})$ becomes larger if interference reduces. In general, for every link l, $\sigma_{l,max}(\tau_k)$ changes as power control continues across the network. The perturbation of $\sigma_{l,max}(\tau_k)$ causes uncertainty in whether $\gamma_l > \frac{1}{\ln 2}\sigma_{l,max}$ holds throughout the process of power control. Specifically, if γ_l is chosen to be close to $\frac{1}{\ln 2}\sigma_{l,max}(\tau_0)$, γ_l may be less than $\frac{1}{\ln 2}\sigma_{l,max}(\tau_k)$ over the subsequent iterations ($k = 1, 2, \cdots$), which may result in that link l resumes transmitting. Therefore, in order to make sure that a low efficiency link is shut down and will not be turned back on, there should be a sufficient margin between γ_l and $\sigma_{l,max}(\tau_0)$, i.e., $\gamma_l \gg \frac{1}{\ln 2}\sigma_{l,max}(\tau_0)$. In Section 3.3, we set $\gamma_l = \infty$ once it is determined that link l should be shut down.

Under certain conditions, the power allocation solution to the optimization problem (3.9) becomes more tractable than general cases. We will look into a special case as follows.

Suppose all inequality constraints in (3.9) are inactive, i.e., $\mu_i = 0$, for $i = 0, 1, \ldots, N_t$, then (3.13) yields

$$z_{l,i} = \frac{1}{\gamma_l \ln 2} - \frac{1}{\sigma_{l,i}} > 0, \;\; i = 1, \ldots, N_t \tag{3.15}$$

Plug (3.15) back into the inactive constraint on transmit power, we have

$$\sum_{i=1}^{N_t} z_{l,i} = \frac{N_t}{\gamma_l \ln 2} - \sum_{i=1}^{N_t} \frac{1}{\sigma_{l,i}} < \bar{p}_l \tag{3.16}$$

which is equivalent to

$$\gamma_l > \frac{N_t}{\left(\bar{p}_l + \sum_{i=1}^{N_t} \sigma_{l,i}^{-1}\right) \ln 2} \tag{3.17}$$

Combining (3.15) and (3.17) yields

$$\frac{N_t}{\left(\bar{p}_l + \sum_{i=1}^{N_t} \sigma_{l,i}^{-1}\right) \ln 2} < \gamma_l < \frac{\sigma_{l,min}}{\ln 2} \tag{3.18}$$

where $\sigma_{l,min} = \underset{i}{min}(\sigma_{l,i})$ and it is assumed that

$$\frac{N_t}{\bar{p}_l + \sum_{i=1}^{N_t} \sigma_{l,i}^{-1}} < \sigma_{l,min} \tag{3.19}$$

(3.19) is a special situation regarding the relationship between transmit power and interference. Based on this assumption, more detailed information can be provided on how the choice of γ_l is related to power allocation. Similar to Fig. 3.1, Fig. 3.2 illustrates strategies for power allocation in link l with respect to different values of γ_l. While in Fig. 3.1 ζ_l cannot be denoted by a closed-form expression, it can be determined in Fig. 3.2 due to the assumption (3.19). The region $\zeta_l < \gamma_l < \sigma_{l,max}/\ln 2$ is further broken down into two intervals by $\sigma_{l,min}/\ln 2$. In the right interval some eigenmodes are not allocated any power, while in the left interval power is allocated to every eigenmode of $\mathbf{Q}_l$ and the exact expression of the power is given.

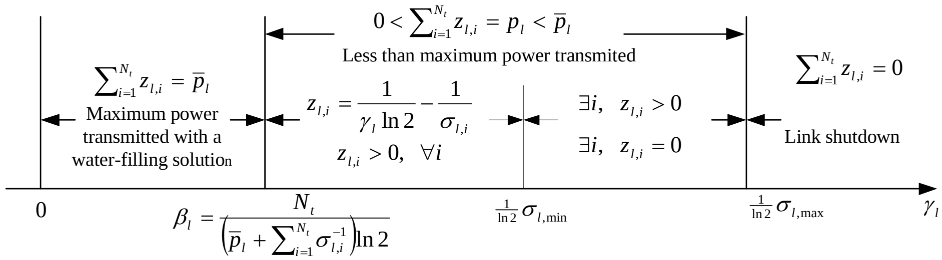

Figure 3.2: Power allocation vs γ_l (a special case)

Fig.3.3 shows a numerical example of power allocation strategy versus the choice of γ_l. In this example, parameters are set as: $N_t = 4$, $\{\sigma_{l,i}\}_{i=1}^4 = (2.0, 1.6, 1.0, 0.4) \cdot \ln 2$ and $\bar{p}_l = 50$. Accordingly, we can find out $\zeta_l = 0.0732$, $\sigma_{l,min} = 0.4$ and $\sigma_{l,max} = 2$. Since $\zeta_l < \sigma_{l,min}$, thus (3.17) is fulfilled. The power allocation results are listed in the figure. All the values agree with analysis shown in Fig.3.2.

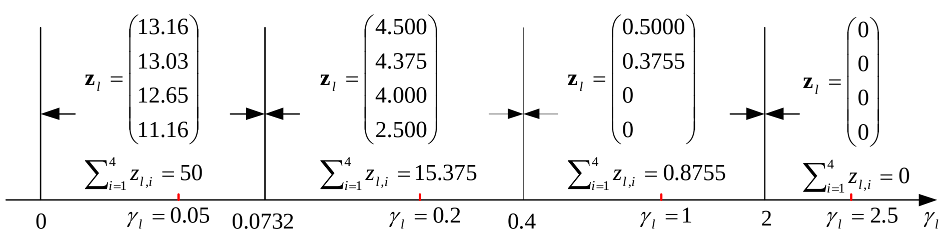

Figure 3.3: Power allocation vs γ_l: a numerical example.

3.2 Game Analysis

In the power control game $T = [\mathbb{L}, \{\mathbb{B}_l\}, \{v_l(\cdot)\}]$, if all links reach an equilibrium point as a result of self-optimizing, power allocation of every link does not change since it is in no link's interest to unilaterally change strategy. The concept of Nash equilibrium provides a predictable outcome of a game, although such an equilibrium is not guaranteed to exist. Next we will investigate this "predictive capability" in the power control game T.

Theorem 5. *A Nash equilibrium exists in the NCG:* $T = [\mathbb{L}, \{\mathbb{B}_l\}, \{v_l(\cdot)\}]$.

Proof. Following the proof of Theorem 2.2, a set of sufficient conditions for the existence of a Nash equilibrium for game T are: for any l,

(a) $\mathbb{B}_l$ is a nonempty, convex and compact subset of a finite Euclidean space.

(b) $v_l(\mathbf{q})$ is continuous in $\mathbf{q}$ and quasi-concave in $\mathbf{z}_l$.

Condition (a) is obviously satisfied for the power control game T. For condition (b), (3.9) indicates that v_l is a continuous function in $\mathbf{q}$. In order to show that the objective function in (3.9) is concave in $\mathbf{z}_l$, we can limit the dimension of $\mathbf{z}_l$ to be 1 because concavity is determined by the behavior of a function on arbitrary lines that intersect its domain [4]. Taking the second-order derivative of v_l with respect to an arbitrary component $z_{l,j}$ yields

$$\frac{\partial^2 v_l}{\partial z_{l,j}^2} = \frac{-\sigma_{l,j}^2}{(1 + \sigma_{l,j} z_{l,j})^2} \leq 0 \tag{3.20}$$

(3.20) shows that v_l is a concave function on $\mathbb{B}_l$, thus both conditions are satisfied, there exists a Nash equilibrium for the power control game $T = [\mathbb{L}, \{\mathbb{B}_l\}, \{v_l(\cdot)\}]$. □

We have shown the existence of a Nash equilibrium of the power control game G. In general, it can be beneficial to have only one equilibrium point, because efficient methods exist to calculate this equilibrium point given uniqueness is guaranteed [12]. However, provable uniqueness of Nash equilibrium is a rare property for non-cooperative games. In [36, 12], some sufficient conditions for the uniqueness of Nash equilibrium were presented, where additional assumptions besides the ones in Theorem 5 were made. Note that these conditions are not necessary for the uniqueness of Nash equilibrium. Thus, while the numerical simulations in the next section may not meet the requirements in [36, 12], a unique Nash equilibrium was found in all simulations.

3.3 Simulation Results with a Soft Shutdown Mechanism

The ray-tracing simulation setup and the network topology are the same as in Chapter 2. We compare the sum data rate and energy efficiency under different methods, namely, GTWS (γ_l assigned to a link based on Equation (3.8)), GGT(General Game Theoretic with soft shutdown, γ_l is assigned to a link based on Equation (3.7)), GP and MUWF. The way to set transmitter power for GTWS, GP and MUWF is the same as in Chapter 2. However, when applying the GGT method, we let each link start with the same transmit power as it does in GTWS. Since low-efficiency links are not necessarily shut down, the total power consumption in the end may be different from that of the other methods. However, the following comparison will still be fair because perturbed SNR values in the GGT method reflect changes in power consumption. In the simulations, the capacity threshold (C_l^t) is set to 2.4 bps/Hz. For the GGT method (Equation (3.7)) and the GTWS method (Equation (3.8)), $\alpha_l = 2$bps.

Fig.3.4 shows the sum data rate of the network for different algorithms and Fig.3.5 demonstrates energy efficiency. The SNR is calculated on the basis of average transmit power across all links. Comparing the two game theoretic approaches, we can see that for SNR <15dB, the two algorithms have similar performances, while at higher SNR, the GGT method undergoes a setback and the GTWS method is substantially better. This observation can be explained from the data in Table 3.1. Link 1 is an inefficient link in the network, so it is turned off by the hard link shut-down mechanism. However, in the GGT approach at high SNR region, by using the normal pricing factor $\gamma_1 = 2/p_o$ instead of setting $\gamma_1 = \infty$, the condition in Theorem 4 is not met, which results in that Link 1 still transmits a significant amount of power and causes interference to other users.

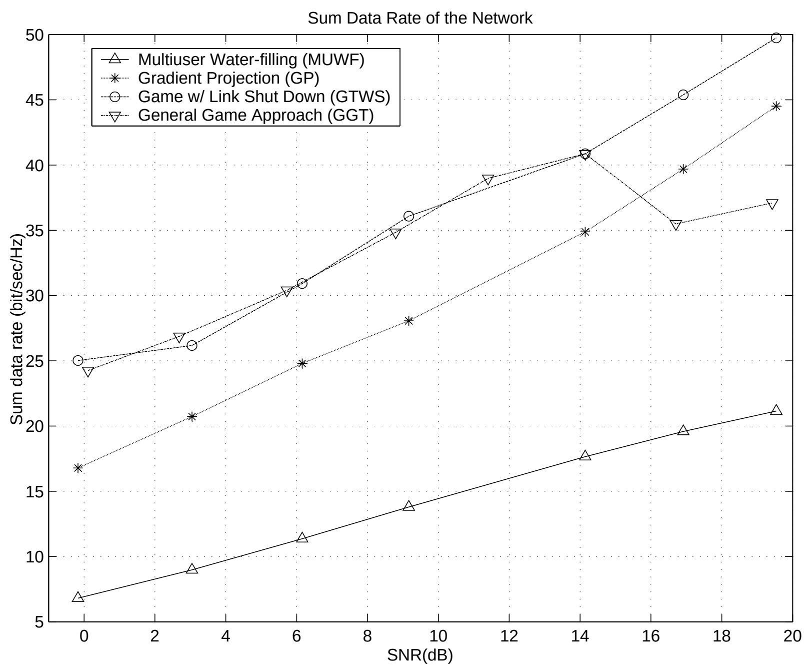

Figure 3.4: Sum data rate of different methods

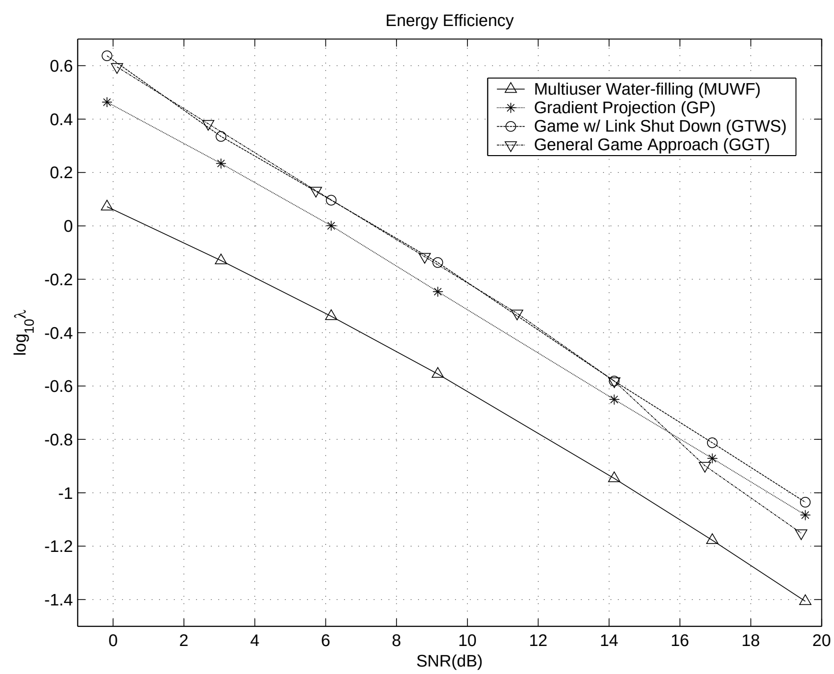

Figure 3.5: Energy efficiency of different methods

Therefore the sum data rate is less than that of the GTWS method. On the other hand, at SNR =14.15dB, the normal pricing factor satisfies the condition in Theorem 4, so it has the same effect as setting $\gamma_l = \infty$ and shuts down inefficient links. This is why at this SNR GGT and GTWS have identical power allocations. For lower SNR values, the performances of the two algorithms are close but not always exactly the same. This is because the fixed γ_l in GGT may not strictly shut down inefficient links and a small amount of power may leak from these links. This power leakage has little effect on the total power consumption or sum data rate.

Data	GTWS (14.15dB)	GGT (14.15dB)	GTWS (16.92dB)	GGT (16.92dB)
L1 Tx Power	0	0.03	0	14.84
L2 Tx Power	42.41	42.41	78.73	79.97
L3 Tx Power	22.71	22.71	40.43	39.75
L4 Tx Power	30.45	30.45	65.40	53.76
L5 Tx Power	19.83	19.83	35.88	32.21
L6 Tx Power	40.45	40.45	74.40	74.78
Sum power	155.85	155.88	294.84	295.31
Sum capacity	40.86	40.86	45.38	35.54

Table 3.1: Link Capacity and transmit power

Comparing the GTWS, GP and MUWF methods in Fig.3.4, we see the GTWS method achieves the highest sum data rate while the MUWF method results in the lowest system capacity. Among the three approaches without the mechanism of hard link shut-down[1], the GP method results in the highest sum data rate. Similar results were reported in Section 3.3, therefore no further discussion is given here.

Fig.3.6 shows how sensitive the system capacity and total power consumption changes with regard γ_l when the GTWS method is used. With a fixed SNR or p_0, it can be seen that a higher γ_l tends to result in a lower total transmit power and consequently a lower sum data rate. This is in agreement with the discussion in Section 3.1.4. It should be

[1]At low SNR (< 15dB), where a link sends out little power, the GGT solution is equivalent to the GTWS solution. We do not compare this SNR region at this point.

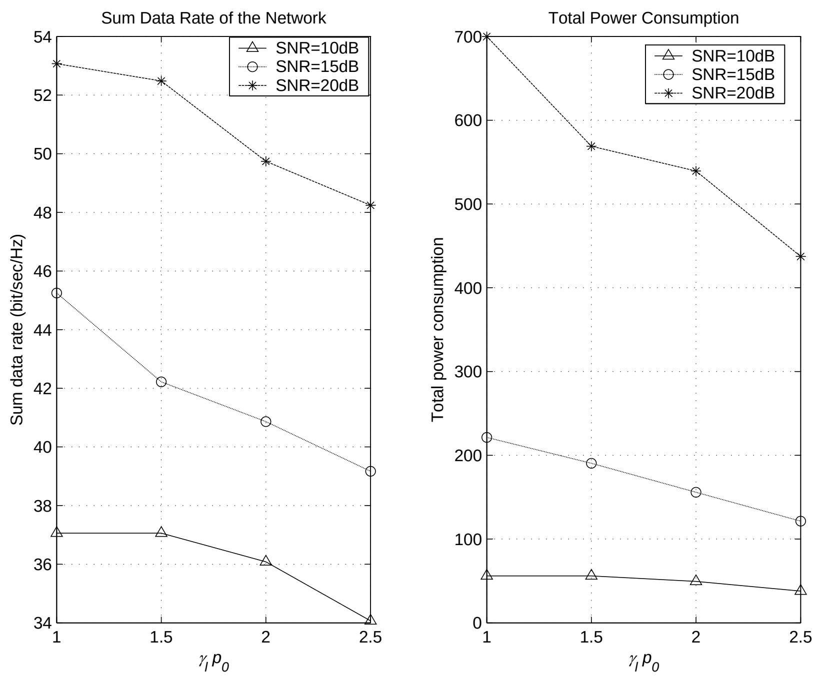

Figure 3.6: System capacity and power consumption vs η_0

noted that as long as its data rate is high enough, a link is not encouraged to choose a smaller γ_l, so that unnecessary transmission of high power is avoided.

3.4 Summary

In this chapter, we designed a new utility function for a power control game with a mechanism of shutting down low-efficiency links. The utility function contains a pricing factor γ_l, which can be tuned to control the amount of transmit power. It was proven that if γ_l is sufficiently large, the solution to the problem of maximizing link utility will be a zero vector, which is equivalent to shutting down the link, thus this mechanism is called soft link shut-down.

4. Power Control for MIMO-OFDM Ad Hoc Networks

Power management for a single-frequency MIMO ad hoc network in a game theoretic approach has been discussed in Chapter 2 and Chapter 3. In this chapter, we will extend this approach to a multiple frequency scenario. With a spectrally efficient modulation technique called Orthogonal Frequency Division Multiplexing, an entire wideband channel are divided into many orthogonal narrowband subchannels (known as subcarriers) to deal with frequency-selective fading. There has been a lot of study on subcarriers assignment and power control in the context of cellular systems. In a SISO-OFDM wireless network with a cellular configuration, for the downlink there is one transmitter (base-station) and multiple receivers (users). Different subcarriers can be allocated to different users so as to provide flexible multiuser access schemes [8, 32]. In most literature, it is assumed a subcarrier can only be assigned to one user at a time because all users communicate with the same base station, thus available subcarriers are partitioned into disjoint subsets, with one subset assigned to one user. In accordance with this restriction, carrier assignment algorithms for broadband wireless networks were presented in [19] where the number of channels (time slots in that paper's scenario) used by each user was minimized. In [31], a suboptimal subchannel allocation algorithm with the assumption of equal power distribution in all subchannels was developed to lower computational complexity. The principle of that method was for each user to use the subchannel with high channel-to-noise ratio as much as possible. Besides looking into subcarrier assignment, rate adaptation with power control has also been investigated rigorously. The approaches are generally in two classes: to minimize overall transmit power given the constraints on the users' data rates or bit error rate [35] and to maximize each user's data rate with a total transmit power constraint [18, 28]. Zhang extended discussion of the above SISO-OFDM formulated cellular network to a MIMO-OFDM cellular network [42] and investigated several issues: exploiting system diversity in frequency, space and user domain, minimizing the overall transmit power and fulfilling each user's QoS requirements including BER and data rate.

In an ad hoc network, there are multiple point-to-point wireless links. Each link is a complete individual communication system. Since every link works in the same frequency band, co-channel interference (CCI) is a key factor to limit the data rate of each link. Therefore, in contrast to a cellular system, resource allocation has to be done with more considerations in interfering links. OFDM systems provide a degree of freedom to allocate power in each subcarrier. If different links use a different subset of available subcarriers, interference experienced by each link may be mitigated due to the fact that a particular subcarrier may not be used by all links. In [20], the problem of distributed subcarrier and bit allocation with power control for a SISO-OFDM ad hoc work was addressed. Although in [42] the MIMO-OFDM network was modeled as a cellular configuration, different users were allowed to transmit at the same subcarrier. The authors took advantage of spatial separability of MIMO systems to avoid or to minimize CCI. In this chapter, we will investigate subcarrier assignment and power allocation for MIMO-OFDM ad hoc networks and apply them in a game-theoretic approach.

4.1 Problem Formulation

4.1.1 Signal Model

We consider an OFDM-based MIMO ad hoc network with no base-station structure. The network consists of a set of point-to-point links denoted by $\mathbb{L} = \{1, 2, ..., L\}$. Let the spectrum be divided into N subcarriers, denoted by $\mathbb{N} = \{1, \cdots, N\}$. Each link undergoes cochannel interference from the other $L-1$ links on the same subcarrier on which transmit nodes are transmitting. Each link uses N_t transmit antennas, N_r receive antennas, and a subset of OFDM subcarriers from $\mathbb{N}$. With OFDM transmission, the frequency-selective fading channel is decoupled into a set of parallel frequency-flat fading channels. The matrix channel between the receive antennas of link l and the transmit antennas of link j on subcarrier k is denoted by $\mathbf{H}_{l,j}^{(k)} \in C^{N_r \times N_t}$.

For all l of the L links, and k of the N subcarriers, the transmitted signal vector,

$\mathbf{x}_l^{(k)} \in C^{N_t \times 1}$ is assumed to be independent across all subcarriers and the receiver array is performing independent single-user detection. The received baseband signal of link l on subcarrier k, $\mathbf{y}_l^{(k)} \in C^{N_r \times 1}$, is given by

$$\mathbf{y}_l^{(k)} = \mathbf{H}_{l,l}^{(k)}\mathbf{x}_l^{(k)} + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}^{(k)}\mathbf{x}_j^{(k)} + \mathbf{n}_l^{(k)} \tag{4.1}$$

where $\mathbf{n}_l^{(k)} \in C^{N_r \times 1}$ is additive white Gaussian noise satisfying

$$E\{\mathbf{n}_l^{(k)}\mathbf{n}_m^{(j)\dagger}\} = \sigma_n^2 \mathbf{I}_{N_r}\delta[l-m, k-j] \tag{4.2}$$

From (4.1), it can be seen that equalization requires application of a narrow-band receiver for each subcarrier k. This channel state information $\mathbf{H}_{l,j}$ can be obtained through channel sounding in the MIMO-OFDM ad hoc network (described in Section 4.4). The transmitted signal $\mathbf{x}_l^{(k)}$ has the covariance matrix $\mathbf{Q}_l^{(k)} = E\{\mathbf{x}_l^{(k)}\mathbf{x}_l^{(k)\dagger}\}$, which denotes power allocation of link l on subcarrier k with the transmit power given by $\mathrm{Tr}(\mathbf{Q}_l^{(k)})$. By aggregating the power allocation matrices over subcarrier, we can denote the power allocation matrix for link l as $\mathbf{Q}_l = diag\{\mathbf{Q}_l^{(k)}\}_{k=0}^{N-1}$ and express the total link transmission power over all subcarriers as $\mathrm{Tr}(\mathbf{Q}_l)$. Clearly, $\mathbf{Q}_l$ determines the power allocation across the transmit antennas and across the OFDM subcarriers.

4.1.2 Mutual Information

If perfect time and frequency synchronization is achieved at the receiver, the multicarrier system can be decoupled into N parallel and independent single carrier systems, and therefore the instantaneous data rate of link l is the sum of data rate at each subcarrier, as given by [9, 3]

$$\begin{aligned} C_l(\mathbf{Q}_1, ..., \mathbf{Q}_L) &= \frac{1}{N}\sum_{k=0}^{N-1} \log_2 \det\left(\mathbf{I} + \mathbf{H}_{l,l}^{(k)}\mathbf{Q}_l^{(k)}\mathbf{H}_{l,l}^{(k)\dagger}(\mathbf{R}_l^{(k)})^{-1}\right) \\ &= \frac{1}{N}\log_2 \det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}) \end{aligned} \tag{4.3}$$

where $\mathbf{R}_l = \mathbf{I} + \sum_{j=1,j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{Q}_j\mathbf{H}_{l,j}^{\dagger}$ is the covariance matrix of the interference-plus-noise of link l. Since $\mathbf{H}_{l,j}$ and $\mathbf{Q}_l$ are block diagonal matrices, so is $\mathbf{R}_l$. The channel matrices $\mathbf{H}_{l,j}$ and $\mathbf{R}_l$ are determined through field measurements described in Section 4.4. Note that the data rate is normalized by N, because N data symbols are transmitted in one OFDM symbol.

Equation (4.3) gives the data rate achievable by a single link l. Similarly, the sum mutual information for the entire wireless network is

$$C(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \sum_{l=1}^{L} \log\det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}). \tag{4.4}$$

4.2 Power Distribution and Subcarrier Assignment

In a MIMO-OFDM ad hoc network, each link may choose to use a portion of all available subcarriers. We denote the set of subcarriers assigned to link l by $\mathbb{D}_l$, with $\mathbb{D}_l \subseteq \mathbb{N}$ and $|\mathbb{D}_l| = 0, 1, \ldots, N$. Thus the number of possible subcarrier assignments for link l is $\binom{N}{0} + \ldots + \binom{N}{N} = 2^N$. Considering there are L links in the network, the total number of subcarrier assignments is 2^{LN}. For a subcarrier k that is not used by link l, i.e., $k \notin \mathbb{D}_l$, no power is transmitted on it and $\mathbf{Q}_l^{(k)} = \mathbf{0}$. Once link l selects a set of subcarriers ($\mathbb{D}_l$) for transmission, its mutual information can be expressed as

$$\begin{aligned} C_l(\mathbf{Q}_1, ..., \mathbf{Q}_L) &= \frac{1}{N} \sum_{k\in\mathbb{D}_l} \log_2 \det\left(\mathbf{I} + \mathbf{H}_{l,l}^{(k)}\mathbf{Q}_l^{(k)}\mathbf{H}_{l,l}^{(k)\dagger}(\mathbf{R}_l^{(k)})^{-1}\right) \\ &= \frac{1}{N} \log_2 \det\left(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\right) \end{aligned} \tag{4.5}$$

Similar to the analysis in Chapter 2 and Chapter 3, link l wishes to maximize an objective function which may be its data rate or capacity-based utility, under the constraint that its transmit power is upper bounded. Without loss of generality, we denote this objective function as $u_l(C_l)$, and formulate an optimization problem for link l as

$$\begin{aligned} \max_{\mathbb{D}_l, \mathbf{Q}_l} \quad & u_l(C_l) \\ s.t. \quad & \mathbb{D}_l \subseteq \mathbb{N} \\ & C_l = \tfrac{1}{N} \log_2 \det \left(\mathbf{I} + \mathbf{H}_{l,l} \mathbf{Q}_l \mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1} \right) \\ & \mathrm{Tr}(\mathbf{Q}_l) \leq \bar{p}_l \ \textit{and} \ \mathbf{Q}_l \geq \mathbf{0} \end{aligned} \tag{4.6}$$

(4.7) is a combinatorial optimization problem. Assigning subcarriers to a link involves integer programming and maximizing the objective function is a nonlinear optimization problem. The objective function is maximized globally over all 2^N subcarrier assignment schemes (assuming all interfering links' resource allocation schemes are fixed), and the corresponding subcarrier allocation and power distribution is the optimal resource allocation solution. However, it is computationally prohibitive to search all possible subcarrier allocation schemes, therefore suboptimal but efficient algorithms need to be derived to solve this problem. In this section, we will discuss power distribution for a fixed subcarrier allocation first, then propose a subcarrier allocation scheme which is suitable for an ad hoc network structure.

4.2.1 Power distribution for a fixed subcarrier allocation

Given a certain subcarrier allocation $\mathbb{D}_l$ for link l, we will look into how to optimize link data rate. If the channel state information is not known at the transmitter, statistically independent data are transmitted from different antennas and different subcarriers, and the total power ($\bar{p}$) is allocated equally across all space-frequency subchannels [27], i.e., the power allocation matrix of link l on any subcarrier k is set to $\mathbf{Q}_l^{(k)} = \mathbf{I}_{N_t} \bar{p}/(N_t|\mathbb{D}_l|)$, and (4.5) is converted to

$$C_l = \frac{1}{N} \sum_{k \in \mathbb{D}_l} \log_2 \left(\det \left(\mathbf{I} + \frac{\bar{p}}{N_t |\mathbb{D}_l|} \mathbf{H}_{l,l}^{(k)} \mathbf{H}_{l,l}^{(k)\dagger} (\mathbf{R}_l^{(k)})^{-1} \right) \right) \tag{4.7}$$

If a no-delay channel feedback mechanism is assumed in place, the transmitters instantly know channel conditions, thus the optimal power allocation to maximize the link data rate is obtained by distributing the total available power according to the water-filling solution. Here the water-filling method is applied to both space and frequency, therefore it is called 2-D water-filling. In the following, we will derive the exact expression for this solution. Denote $\mathbf{H}_{l,l}^{(k)\dagger}(\mathbf{R}_l^{(k)})^{-1}\mathbf{H}_{l,l}^{(k)} = \mathbf{U}_l^{(k)}\mathbf{\Sigma}_l^{(k)}\mathbf{U}_l^{(k)\dagger}$ as an eigenvalue decomposition of $\mathbf{H}_{l,l}^{(k)\dagger}(\mathbf{R}_l^{(k)})^{-1}\mathbf{H}_{l,l}^{(k)}$ for any $k \in \mathbb{N}$. If $k \in \mathbb{D}_l$, the receive node receives signal of interest as well as interference on subcarrier k and $\mathbf{\Sigma}_l^{(k)} \neq \mathbf{0}$. On the other hand, if $k \notin \mathbb{D}_l$, then $\mathbf{\Sigma}_l^{(k)} = \mathbf{0}$ and the corresponding eigenmatrix $\mathbf{U}_l^{(k)}$ can be any unitary matrix. For the sake of simplicity, set $\mathbf{U}_l^{(k)} = \mathbf{I}$. Aggregating all the eigenvalue decompositions into a block diagonal matrix, we see

$$\begin{aligned}\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l} &= \begin{bmatrix} \mathbf{U}_l^{(0)}\mathbf{\Sigma}_l^{(0)}\mathbf{U}_l^{(0)\dagger} & & & \\ & \mathbf{U}_l^{(1)}\mathbf{\Sigma}_l^{(1)}\mathbf{U}_l^{(1)\dagger} & & \\ & & \ddots & \\ & & & \mathbf{U}_l^{(N-1)}\mathbf{\Sigma}_l^{(N-1)}\mathbf{U}_l^{(N-1)\dagger} \end{bmatrix} \\ &= \mathbf{U}_l\mathbf{\Sigma}_l\mathbf{U}_l^{\dagger} \end{aligned} \tag{4.8}$$

where $\mathbf{U}_l = diag\{\mathbf{U}_l^{(k)}\}_{k=0}^{N-1}$ is unitary and $\mathbf{\Sigma}_l = diag\{\mathbf{\Sigma}_l^{(k)}\}_{k=0}^{N-1}$ is diagonal. In fact, (4.8) is an eigenvalue decomposition of the augmented block diagonal matrix $\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l}$. The eigenvalues and corresponding eigenvectors of the augmented matrix are the concatenation of the eigenvalues and eigenvectors of each matrix block, respectively. By expanding $\mathbf{\Sigma}_l$ as $diag\{\sigma_{l,1}^{(0)}, \cdots, \sigma_{l,N_t}^{(0)}, \cdots, \sigma_{l,1}^{(N-1)}, \cdots, \sigma_{l,N_t}^{(N-1)}\}$ and using Hadamard's inequality [23], it is easy to see that the optimal power allocation matrix $\mathbf{Q}_l$ is a pure diagonal matrix with nonnegative elements, as given by

$$\mathbf{Q}_l = diag\{z_{l,1}^{(0)}, \cdots, z_{l,N_t}^{(0)}, \cdots, z_{l,1}^{(N-1)}, \cdots, z_{l,N_t}^{(N-1)}\} \tag{4.9}$$

where $z_{l,1}^{(k)}, \cdots, z_{l,N_t}^{(k)}$ are the power allocated to the eigenmodes of subcarrier k. Plug

(4.9)(4.8) into (4.3) and the expression of mutual information is simplified as

$$C_l = \frac{1}{N} \sum_{k \in \mathbb{D}_l} \sum_{i=1}^{N_t} \log_2 \left(1 + z_{l,i}^{(k)} \sigma_{l,i}^{(k)}\right) \tag{4.10}$$

with the power constraint $\text{Tr}(\mathbf{Q}_l) \leq \bar{p}_l$. This is a standard 1-D water-filling problem and the solution is $z_{l,i}^{(k)} = (\mu - 1/\sigma_{l,i}^{(k)})^+$, where μ is a constant chosen so that $\sum_{k \in \mathbb{D}_l} \sum_{i=1}^{N_t} z_{l,i}^{(k)} = \bar{p}_l$.

Equation (4.3) gives the data rate achievable by a single link l. Similarly, the sum mutual information for the entire wireless network is

$$C(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \sum_{l=1}^{L} \log \det(\mathbf{I} + \mathbf{H}_{l,l} \mathbf{Q}_l \mathbf{H}_{l,l}^{\dagger} \mathbf{R}_l^{-1}). \tag{4.11}$$

The gradient projection method can also be used for optimizing the sum data rate of the OFDM-based MIMO ad hoc network. The power allocation matrix for each link is an augmented block diagonal matrix as described before. For any link l, the transmit node calculates the gradient of sum mutual information with respect to its own power allocation $\mathbf{Q}_l$, and then update its transmission strategy. Every link follows the same course of actions iteratively until the sum data rate converges.

4.2.2 A Subcarrier Assignment Scheme

If frequency reuse is not allowed (typically in a cellular-structure multiuser OFDM system), the batch of subcarriers used by a user does not overlap with that of another user. This idea can be directly applied to a MIMO-OFDM ad hoc network. Figure 4.1 illustrates a fixed subcarrier assignment scheme in which each link uses a batch of continuous subcarriers. The subcarrier numbering is in accordance with IEEE802.11a standard, where only subcarriers $[2:27] \bigcup [39:64]$ are used for data transmission.

The advantage of non-overlapping subcarrier assignment is that co-channel interference is avoided. However, since every link only uses a small portion of frequency spectrum, its data rate can be very low. The water-filling solution (4.10) shows that the link data rate is the sum mutual information of each single-frequency MIMO system, while the data rate

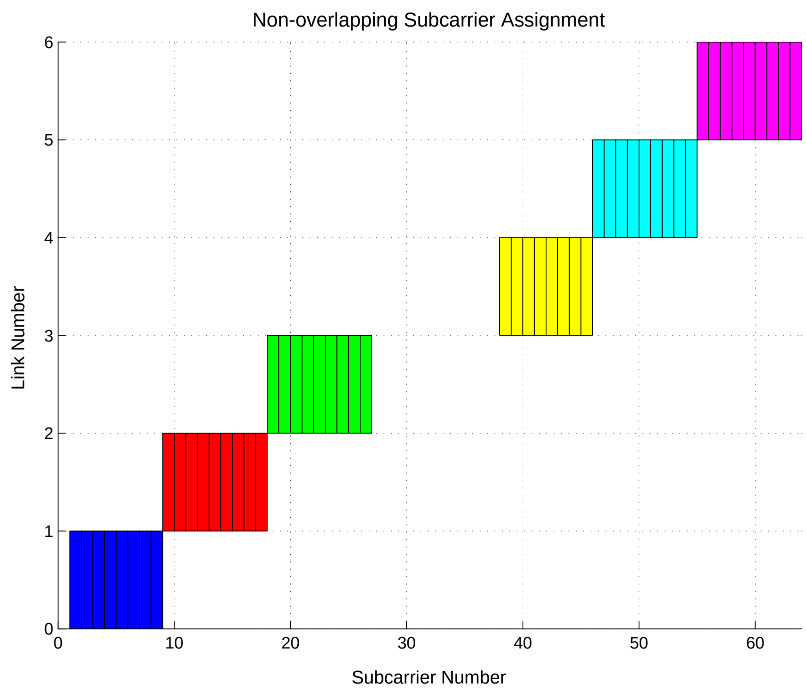

Figure 4.1: Non-overlapping subcarrier assignment

of a single-frequency MIMO system is the sum mutual inforamtion of the virtual channels corresponding to the interfered channel's eigenmodes. If we only look at an arbitrary eigenmode i related to subcarrier k, its mutual information is

$$C_{l,i}^{k} = \log_2(1 + z_{l,i}^{(k)} \sigma_{l,i}^{(k)}) \tag{4.12}$$

which is equivalent to

$$z_{l,i}^{(k)} = \frac{2^{C_{l,i}^{(k)}} - 1}{\sigma_{l,i}^{(k)}} \tag{4.13}$$

The above equations indicate that the transmit power over an eigenmode on a subcarrier increases exponentially with the data rate it achieves. In other words, the price for demanding higher rate is transmitting exponentially more power. Therefore, given the total transmit power upper bounded, the multiple-antenna transmitter tends to allocate power over a large number of subcarriers. Each subcarrier may be allocated a small amount of power, however, the sum rate will be high. To coarsely show this effect quantitatively, we use a simplified illustrative example as follows. Suppose $\sigma_{l,i}^{(k)} = \sigma_0$ is a constant for all k and i, thus the water-filling solution (4.10) corresponds to equal power allocation, i.e., $z_{l,i}^{(k)} = \bar{p}/N_t|\mathbb{D}_l|$. Let $\alpha = |\mathbb{D}_l|/N$ be the proportion of subcarriers in $\mathbb{N}$ that are assigned to $\mathbb{D}_l$, then the ratio of data rate $C_l(\mathbb{D}_l)$ to $C_l(\mathbb{N})$ is

$$\beta = \frac{\alpha N N_t \log_2 \left(1 + \frac{\bar{p}_l \sigma_0}{\alpha N N_t}\right)}{N N_t \log_2 \left(1 + \frac{\bar{p}_l \sigma_0}{N N_t}\right)} = \frac{\alpha \log_2 \left(1 + \frac{\rho}{\alpha}\right)}{\log_2(1 + \rho)} \tag{4.14}$$

where $\rho = \bar{p}_l \sigma_0 / N N_t$ is signal-to-noise ratio on a per antenna/subcarrier basis. Figure 4.2 plots β versus ρ with α as a parameter. It can be seen that the fewer number of subcarriers are picked for $\mathbb{D}_l$, the lower the data rate of that link.

In a MIMO-OFDM ad hoc network with point-to-point links, if all links choose to spread power to as many as subcarriers as possible, the frequency reuse factor will be high, which may result in severe co-channel interference between these links. In order for

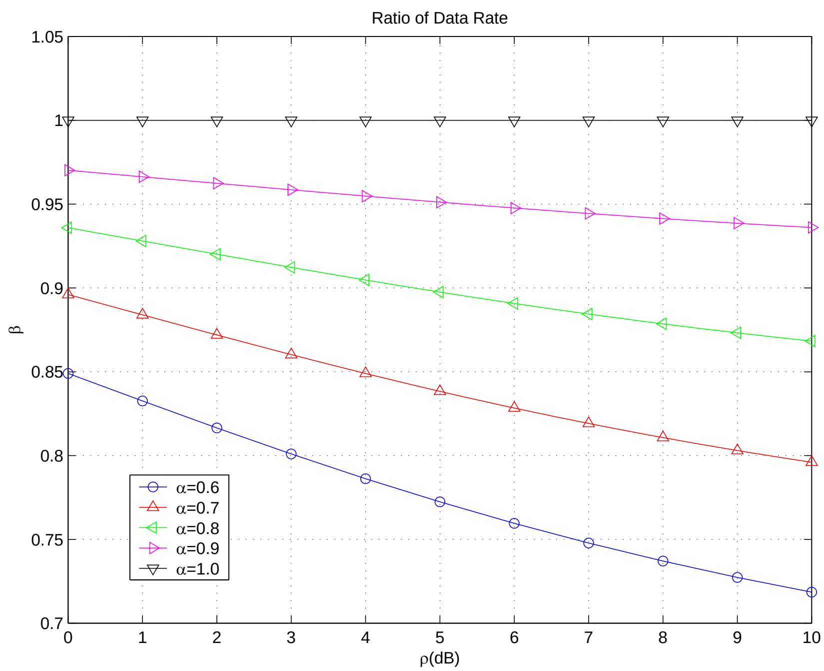

Figure 4.2: Ratio of data rate $(C_l(\mathbb{D}_l)/C_l(\mathbb{N}))$

a link to counter interference, transmit power will be increased. Obviously this situation is not ideal for improving energy efficiency. Another strategy is for each link to use a limited number of subcarriers but not to increase its total power consumption. In this case, the power allocated to each assigned subcarrier is higher as opposed to using all subcarriers. Although Figure 4.2 indicates a lower link rate for using less subcarriers, the assumption ($\sigma_{l,i}^{(k)} = \sigma_0, \forall k, i$) is unrealistic for a MIMO-OFDM network where dynamic subcarrier assignment is allowed. Therefore, by taking into accounts some practical factors, we may not necessarily prefer to use the most subcarriers available. To be specific, a wideband channel has different fading characteristics from one subcarrier to another. Higher data rate is achieved on a subcarrier with higher carrier-to-noise ratio[1] than on one with lower ratio, given the same amount of power is transmitted. Therefore, allocating most or all power into "good" subcarriers can result in data rate close to the optimal solution. Furthermore, since each link may undergo independent channel fading from other links, the set of transmit frequencies chosen for one link may be substantially different from another, which will reduce co-channel interference in the network and may improve data rate for some of the links.

Following the heuristic above, we propose an algorithm which selects a set of subcarriers for each link so that a high data rate can be achieved and co-channel interference can be mitigated. First, each link calculates its independence water-filling capacity assuming all N subcarriers are used. We denote this capacity for link l as $C_l(\mathbb{N})$. This is the maximum data rate that link l can achieve since no interference is taken into consideration. Each link then identifies the channel condition in each subcarrier, which will be exploited to determine if a subcarrier is selected or not. In this step, each link allocates equal power to every subcarrier ($\bar{p}_l/N$) and apply independent water-filling method to find out $C_l^{(k)}$ for every k. As the transmit power is the same across all subcarriers, $C_l^{(k)}$ indicates the MIMO channel condition on the frequency k. We then sort $C_l^{(k)}(k = 0, \ldots, N)$ in descending order, which will later facilitate choosing subchannels in the same fashion.

[1]Carrier to noise ratio is defined as $|h|^2/\sigma_n^2$, where h is channel coefficient.

Next, link l should decide which subcarriers to put into $\mathbb{D}_l$. Two considerations need to be taken here: (1) Given fixed interference and the same maximum transmit power, allocating power over $\mathbb{D}_l$ will result in a less data rate than over $\mathbb{N}$. This is because to maximize the same objective function (C_l), the feasible region of the former method is no larger than that of the latter[2], thus its solution cannot outperform that of the latter. In order that its data rate does not drop too much due to using less subcarriers, link l may require that $C_l(\mathbb{D}_l)$ is no less than $\beta_l C_l(\mathbb{N})$, where $\beta_l \in [0, 1]$ is a proportional factor. The choice of β_l is mainly affected by the type of service and the channel condition. If the channel is good with a high $C_l(\mathbb{N})$, but very high data is not required by link l, then β_l can be set to a relatively low value. In contrast, if a link demands as high data rate as possible, β_l should be chosen to be close to 1. (2) $|\mathbb{D}_l|$ should be minimized as long as $C_l(\mathbb{D}_l) \geq \beta_l C_l(\mathbb{N})$ is satisfied. A high $|\mathbb{D}_l|$ means high frequency reuse factor. To mitigate co-channel interference, each link is encouraged to use the minimum number of subcarriers once its targeted data rate is reached. In the following, the proposed subcarrier assignment algorithm is described in detail.

Algorithm 4.1: Subcarrier Assignment

1. Calculate independent water-filling capacity.

1.1 Each link calculates its independent water-filling capacity ($C_l(\mathbb{N})$) over all N subcarriers, with the constraint $\mathrm{Tr}(\mathbf{Q}_l) \leq \bar{p}_l$.

1.2 For every subcarrier k, find its water-filling capacity, with the constraint $\mathrm{Tr}(\mathbf{Q}_l^{(k)}) \leq \bar{p}_l/N$. Sort $C_l^{(k)}$ $(k = 0, \ldots, N-1)$ in descending order. Denote the sorted sequence as $\{C_l^{(s_i)}\}_{i=0}^{N-1}$ with $s_i = k$.

2. Admit subcarriers into $\mathbb{D}_l$.

2.1 Based on the channel conditions and data rate requirement, each link selects a capacity proportional factor β_l with $0 \leq \beta_l \leq 1$.

2.2 Initialize $i = 0$, $\mathbb{D}_l = \varnothing$ and $C_l(\mathbb{D}_l) = 0$;

[2]Given $\mathbb{D}_l \subseteq \mathbb{N}$, we know if $\mathbb{D}_l = \mathbb{N}$, the two problems are exactly the same, so are the solutions. If $\mathbb{D}_l \subset \mathbb{N}$, then $\exists k \in \mathbb{N}$ but $k \notin \mathbb{D}_l$, and the optimization problem in $\mathbb{D}_l$ contain constraint $\mathrm{Tr}(\mathbf{Q}_l^{(k)}) = 0$, which are not present in the optimization problem in $\mathbb{N}$.

Do while $C_l(\mathbb{D}_l) < \beta_l C_l(\mathbb{N})$

$\mathbb{D}_l = \mathbb{D}_l \bigcup \{s_i\}$;

Calculate $C_l(\mathbb{D}_l)$ using independent water-filling, with the constraint $\mathrm{Tr}(\mathbf{Q}_l) \leq \bar{p}_l$;

$i = i + 1$;

End

4.2.3 Illustrative Example

A simple example will be shown here to illustrate the outcome of the subcarrier assignment algorithm. We took measurements of the channel responses of each link in a six-link MIMO-OFDM ad hoc network. For each link, the channel matrices for all 52 subcarriers were acquired. For the sake of simplicity, we assume each link chooses the same β_l, i.e., $\beta_l = \beta, \forall l$. Table 4.1 lists the number of subcarriers assigned to each link for different capacity proportional factor β. Figure 4.3-Figure 4.6 illustrate exactly which subcarriers are assigned to a link with β as a parameter. We can see from Table 4.1 that more subcarriers are used when β gets larger. This is expected because $C_l(\mathbb{D}_l)$ tends closer to $C_l(\mathbb{N})$ when $\mathbb{D}_l \longrightarrow \mathbb{N}$. It is shown in Figure 4.3-Figure 4.6 that co-channel interference can be mitigated to a certain degree since different links are assigned different subsets of available carriers. For instance, if we compare the frequency reuse between link 1 and link 2, we see that the numbers of subcarriers that are shared by both links are 10, 22, 36 and 44 for β=0.7,0.8,0.9 and 0.95, respectively. The corresponding ratios of number of unshared subcarriers to $|\mathbb{D}_1|$ are $68\%, 41\%, 18\%$ and 8%.

$\lvert\mathbb{D}_l\rvert$	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
$\beta = 0.70$	31	31	27	30	31	30
$\beta = 0.80$	37	37	33	36	37	37
$\beta = 0.90$	44	44	41	43	44	45
$\beta = 0.95$	48	48	45	48	48	48

Table 4.1: Number of subcarreirs assigned for each link

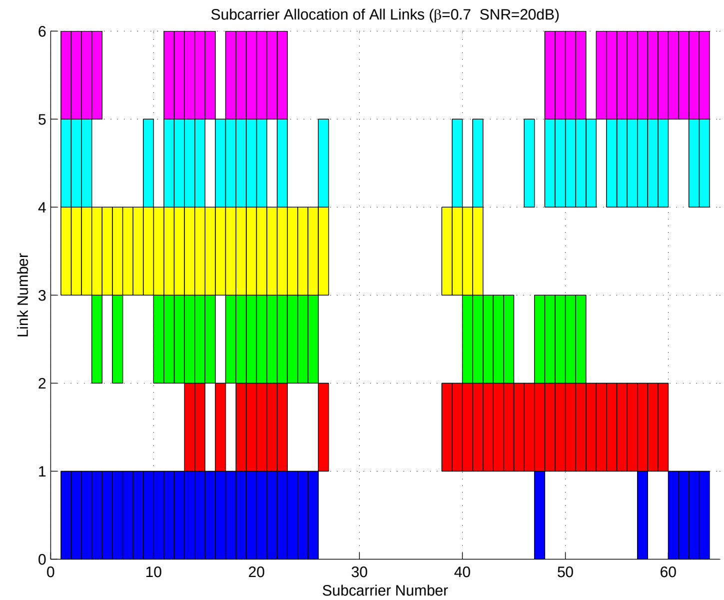

Figure 4.3: Subcarrier allocation of all links ($\beta = 0.7$, SNR=20dB)

4.3 Resource Allocation in MIMO-OFDM Ad Hoc Networks - A Game Theoretic Approach

In Chapter 3, power management in a single-frequency MIMO ad hoc network was built into a game. By choosing a pricing factor γ_l, link l is able to control whether to send the maximal power or to shut itself down. Accordingly, energy efficiency across the network can be improved. In a MIMO-OFDM ad hoc network as we are discussing in this chapter, power control and subcarrier assignment are two dimensions of resource allocation. However, assigning no subcarriers to a link is equivalent to sending no power on that link. Therefore, if a link can adaptively change the transmit power on each subcarrier, the two-dimensional resource allocation problem can be simplified to a power control problem. In this section, we will extend the game theoretic formulation to power management and subcarrier assignment in a MIMO-OFDM ad hoc network.

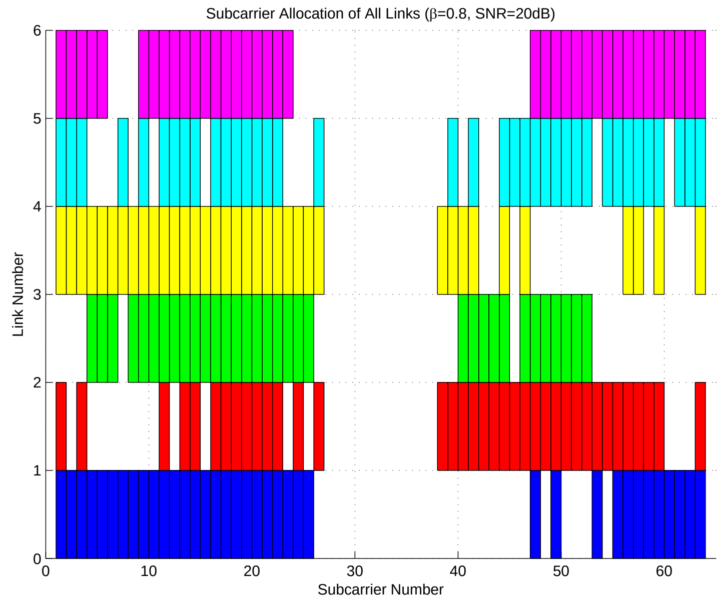

Figure 4.4: Subcarrier allocation of all links ($\beta = 0.8$, SNR=20dB)

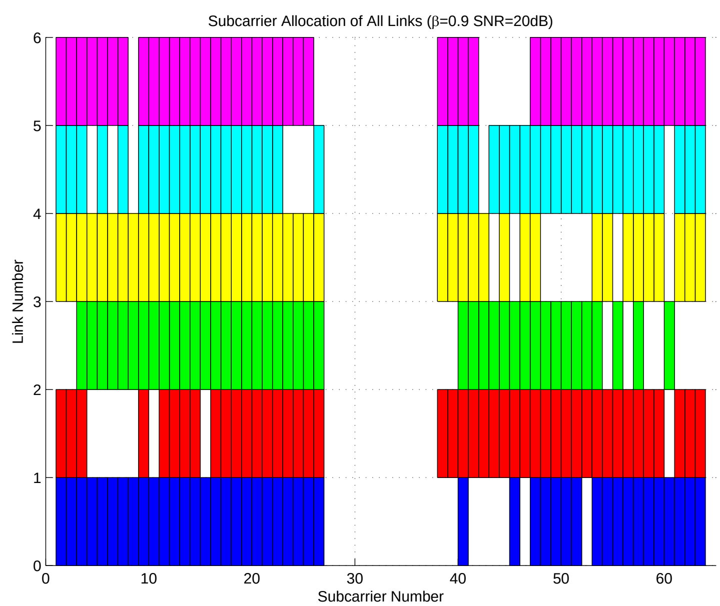

Figure 4.5: Subcarrier allocation of all links ($\beta = 0.9$, SNR=20dB)

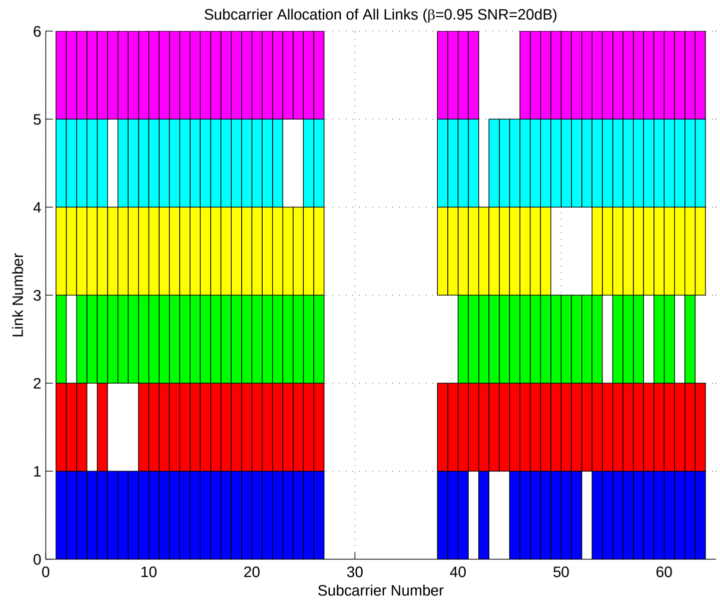

Figure 4.6: Subcarrier allocation of all links ($\beta = 0.95$, SNR=20dB)

4.3.1 Utility Function and Game Formulation

Similar to the utility function design in Chapter 3, the net utility of a link in a MIMO-OFDM ad hoc network is related to its data rate and power consumption. The former can be calculated by (4.5), which is the sum of mutual information across all subcarriers. Power consumption is treated again as a pricing term here. Since the amount of power is adaptively allocated to each subcarrier, we assign a different pricing factor $\gamma_l^{(k)}$ to each subcarrier. Thus, the net utility of the lth link is expressed as

$$w_l = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{i=1}^{N_t} \log_2 \left(1 + z_{l,i}^{(k)} \sigma_{l,i}^{(k)}\right) - \sum_{k=0}^{N-1} \gamma_l^{(k)} \sum_{i=1}^{N_t} z_{l,i}^{(k)} \tag{4.15}$$

where $\gamma_l^{(k)}$ is a nonnegative scaling factor with the unit bps/Hz/Watt. The maximum data rate is achieved when the highest power is transmitted, but the pricing term discourages the node from sending maximum power, so the net utility (4.15) balances benefit and cost, and it is the objective function that each link tries to maximize.

Assuming there is no cooperation between MIMO-OFDM links in an ad hoc network, we model interactions among those links as a non-cooperative game (NCG): $O = [\mathbb{L}, \{\mathbb{E}_l\}, \{w_l(\cdot)\}]$, where $\mathbb{L}$ is the set of links, $\mathbb{E}_l=\{\mathbf{z}_l \in \mathbb{R}_+^{N_t N} | \sum_{k=0}^{N-1}\sum_{i=1}^{N_t} z_{l,i}^{(k)} \leq \bar{p}_l\}$ is the set of power allocation actions and $w_l(\cdot)$ is the utility function of link l. Let $\mathbf{q}(\tau_k) = (\mathbf{z}_1(\tau_k), ..., \mathbf{z}_L(\tau_k))$ denote the power allocation strategies of all links at τ_k and $\mathbf{q}_{-l}(\tau_k)$ denote the vector consisting of elements of $\mathbf{q}(\tau_k)$ without the l^{th} link. For any link l at time τ_k, the transmitter tries to find $\mathbf{z}_l(\tau_k) \in \mathbb{E}_l$, such that for any other $\mathbf{z}_l^{'}(\tau_k) \in \mathbb{E}_l$, $w_l(\mathbf{z}_l(\tau_k), \mathbf{q}_{-l}(\tau_k)) \geq w_l(\mathbf{z}_l^{'}(\tau_k), \mathbf{q}_{-l}(\tau_k))$. Each link tries to calculate its own optimum power allocation $\mathbf{z}_l$ which maximizes its net utility.

4.3.2 Relation between subcarrier assignment and choice of $\gamma_l^{(k)}$

In the game O, link l tries to solve the following convex optimization problem

$$\begin{aligned} \max_{\mathbf{z}_l} \quad & \frac{1}{N}\sum_{k=0}^{N-1}\sum_{i=1}^{N_t} \log_2\left(1 + z_{l,i}^{(k)}\sigma_{l,i}^{(k)}\right) - \sum_{k=0}^{N-1}\gamma_l^{(k)}\sum_{i=1}^{N_t} z_{l,i}^{(k)} \\ s.t. \quad & \sum_{k=0}^{N-1}\sum_{i=1}^{N_t} z_{l,i}^{(k)} \leq \bar{p}_l \\ & z_{l,i}^{(k)} \geq 0, \ \forall i, k \end{aligned} \tag{4.16}$$

We analyze the optimization problem with Lagrange multiplier theory. The Lagrangian function in a minimization formulation is given as

$$\begin{aligned} L(\mathbf{z}_l, \mathbf{u}) = & \sum_{k=0}^{N-1}\gamma_l^{(k)}\sum_{i=1}^{N_t} z_{l,i}^{(k)} - \frac{1}{N}\sum_{k=0}^{N-1}\sum_{i=1}^{N_t}\log_2\left(1 + z_{l,i}^{(k)}\sigma_{l,i}^{(k)}\right) \\ & + \mu_0\left(\sum_{k=0}^{N-1}\sum_{i=1}^{N_t} z_{l,i}^{(k)} - \bar{p}_l\right) - \sum_{k=0}^{N-1}\sum_{i=1}^{N_t}\mu_i^{(k)} z_{l,i}^{(k)} \end{aligned} \tag{4.17}$$

where $\mathbf{u} = (\mu_0, \mu_1^{(0)}, \ldots, \mu_{N_t}^{(N-1)})$ is the Lagrange multiplier vector and $\sigma_{l,1}^{(0)}, ..., \sigma_{l,N_t}^{(N-1)}$ are constants due to the assumption that all the other links' power allocations ($\mathbf{q}_{-l}$) are given. If a $\mathbf{z}_l$ is a local maximum of (4.16), from KKT necessary condition we know there exists a unique **u**, such that

$$\frac{\partial L}{\partial z_{l,i}^{(k)}} = \gamma_l^{(k)} - \frac{\sigma_{l,i}^{(k)}}{(1+\sigma_{l,i}^{(k)} z_{l,i}^{(k)})N \ln 2} + \mu_0 - \mu_i^{(k)} = 0,\ i = 1, ..., N_t,\ k \in \mathbb{N} \quad (4.18)$$

$$\mu_i^{(k)} \geq 0,\ i = 1, \ldots, N_t,\ k \in \mathbb{N} \quad (4.19)$$

$$\mu_0 \geq 0 \quad (4.20)$$

Following the proof of Theorem 3.2, we can see that if $\gamma_l^{(k)} > \frac{1}{N \ln 2} \max_i \left(\sigma_{l,i}^{(k)}\right)$, the solution to (4.18)-(4.20) will be $z_{l,i}^{(k)} = 0$, $\forall i$. The physical meaning of this observation is that no power is transmitted on subcarrier k if the pricing factor of that particular subcarrier is sufficiently large.

In Section 4.2.2 we proposed a subcarrier assignment scheme which allowed a link to use only a subset ($\mathbb{D}_l$) of available subcarriers ($\mathbb{N}$). For a subcarrier $k \notin \mathbb{D}_l$, the subsystem at that carrier was shut down. The net utility function (4.15) facilitates this subcarrier assignment scheme by using a tunable pricing factor $\gamma_l^{(k)}$ for each subcarrier. If link l voluntarily chooses not to use a particular subcarrier k, it can set a sufficiently large value to $\gamma_l^{(k)}$ so that that subcarrier is shut down for link l[3]. Another interesting observation about the function (4.15) is that it allows link l to take or drop a subcarrier in a soft and adaptive fashion. Suppose that link l receives increasing interference at subcarrier k, thus $\{\sigma_{l,i}^{(k)}\}_{i=1}^{N_t}$ decrease. Once $\gamma_l^{(k)} > \frac{1}{N \ln 2} \max_i \left(\sigma_{l,i}^{(k)}\right)$ is fulfilled, transmit power on that subcarrier is reduced to zero and that subcarrier drops out of $\mathbb{D}_l$. Conversely, when link l detects less interference on a certain subcarrier which is not in $\mathbb{D}_l$ yet, it may admit that subcarrier if $\gamma_l^{(k)} < \frac{1}{N \ln 2} \max_i \left(\sigma_{l,i}^{(k)}\right)$ is satisfied.

4.3.3 Game Algorithm

In this section, we propose an algorithm for subcarrier allocation and power distribution in a game-theoretic approach. First, following Algorithm 4.1, each link determines

[3]Here a sufficiently large $\gamma_l^{(k)}$ means that$\gamma_l^{(k)} > \frac{1}{N \ln 2} \max_i \left(\sigma_{l,i}^{(k)}\right)$ holds, so that there is no power allocated to subcarrier k for link l.

its set of subcarriers to use. Second, for any link l, it selects pricing factor $\gamma_l^{(k)}$ for all $k \in \mathbb{D}_l$ and set a large value to $\gamma_l^{(k)}$ if $k \notin \mathbb{D}_l$. Link l calculates the optimum p_l such that the net utility function is maximized. This p_l corresponds to a $\mathbf{z}_l$ which is determined by maximizing (4.15).

Algorithm 4.2: Game-theoretic approach to power control in MIMO-OFDM ad hoc networks

1 Each link determines its set of subcarriers to use.

Each link selects a capacity proportional number and determines a set of subcarriers ($\mathbb{D}_l$) using Algorithm 4.1.

Set k = 0;

Let $\mathbf{q}(\tau_0)$ be the power allocation that each link has when it finds its $\mathbb{D}_l$.

2 Update power allocations:

Let $k = k+1$. For all links $l \in \mathbb{L}$, given power allocation vector $\mathbf{q}(\tau_{k-1})$, compute $\mathbf{z}_l(\tau_k) = \arg\max_{\mathbf{z}_l \in \mathbb{E}_l} w_l(\mathbf{z}_l, \mathbf{q}_{-l}(\tau_{k-1}))$. Repeat step 2 until the net utility of every link converges.

4.3.4 Game Analysis

The net utility function (4.5) in this chapter is similar to (3.4) in Chapter 3. The only difference is that (4.5) extended the power allocation vector on a single-frequency to the power allocation vector across all subcarriers. Mathematically, more variables are introduced into optimization problems, but both the structure of the net utility function and the linear constraints remain the same, hence the properties of the feasible set ($\mathbb{E}$ versus $\mathbb{B}$) and the concavity of the net utility function (w_l versus v_l) do not change. Therefore, the existence of Nash equilibrium of Game O can be shown by following the same procedure of proofs as in Section 3.2.

4.4 Numerical Results Based on Channel Measurements

We took channel measurements of MIMO-OFDM indoor local area network links on the 3^{rd} floor of the Bossone building on the campus of Drexel University. The network topology that we tested had six links and twelve nodes. The nodes were scattered throughout the area and contained a mixture of long-distance versus short-distance, LOS versus NLOS communications links. The layout of node locations is shown in Figure 4.7. Nodes number from 1 to 6 are transmit nodes and all the other nodes are receive nodes. For each transmit location, the channel was measured at every receive location. Therefore we could analyze the channels of interest ($\mathbf{H}_{l,l}$) and interfering ($\mathbf{H}_{l,j}$) links. The data has been made available online at *http://www.ece.drexel.edu/wireless.*

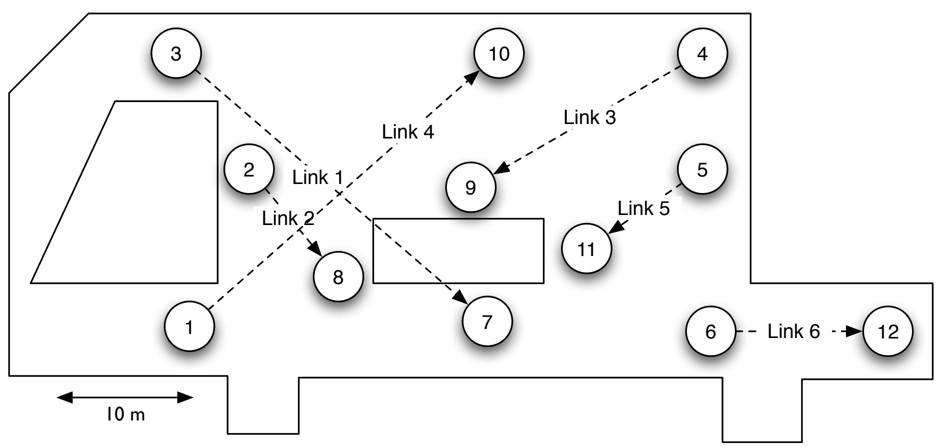

Figure 4.7: Indoor measurement testing environment and node locations

Figure 4.8 shows the data rate of all links versus SNR when game theoretic method (GGT) is used. The calculated capacity flattens because as SNR increases for the link of interest, the interference also increases. Link 6 has much greater capacity than the other links because the receiver is affected minimally by the other links. Link 4 and Link 1 are practically shut down because their data rates are close to zero. This is due to the presence of many other transmitting nodes as well as the lengths of these two links.

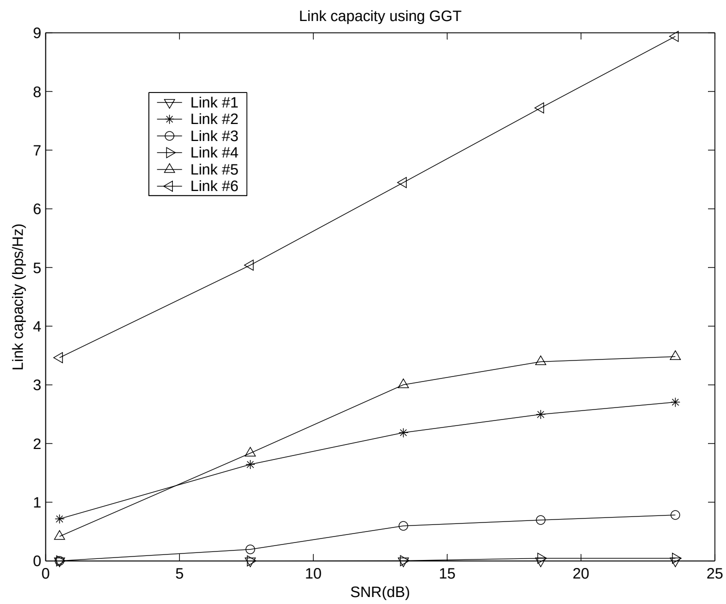

Figure 4.8: Link data rate using game theoretic technique

Figure 4.9 shows the data rate of all links versus SNR when multiuser water-filling (MUWF) technique is used. In order to make a fair comparison with the game theoretic method, we fix the total power consumption of the network, which has been determined by the GGT method, and allocate power equally to all six links in the network. It can be seen that for links with good channel conditions (Link 6 and 5), their data rates are lower than those in the GGT method. For links with poor channel conditions (Link 4 and 1), their data rates are higher than those in the GGT method. The sum data rates of the two techniques are shown in Figure 4.10. As SNR increases, the GGT method outperforms the MUWF method by a larger margin. This is because in the MUWF method links with low data rates send out the same amount of power and cause considerable interference to links with high data rate. While in the GGT method, inefficient links are practically shut down, thus existing links are allocated higher power and experience less interference.

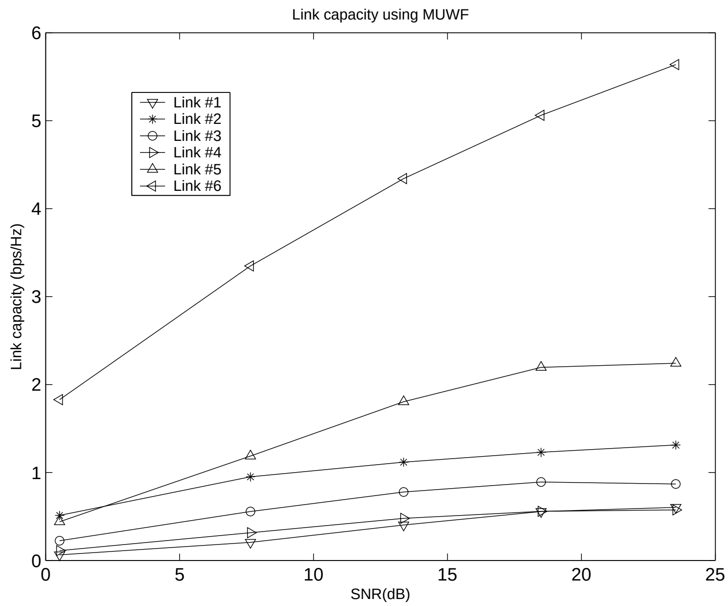

Figure 4.9: Link data rate using multiuser water-filling

When the GGT method is used, Figure 4.11 and Figure 4.12 demonstrate the sum data rate and energy efficiency versus different β values, respectively. Figure 4.11 shows that for the measured topology, increasing β increases sum data rate for low SNR region. At high SNR, increasing β may cause high interference and reduce sum data rate. It can be seen from 4.12 that a low β results in high energy efficiency. Increasing β results in decreased network energy efficiency. Therefore, given that the link data rate meets a certain requirement, it is advisable to choose a small β.

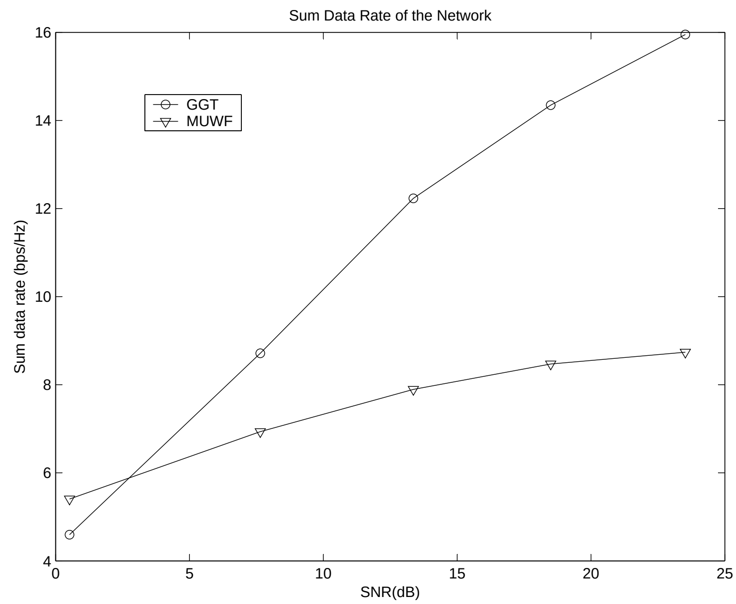

Figure 4.10: Sum data rate of GGT and MUWF

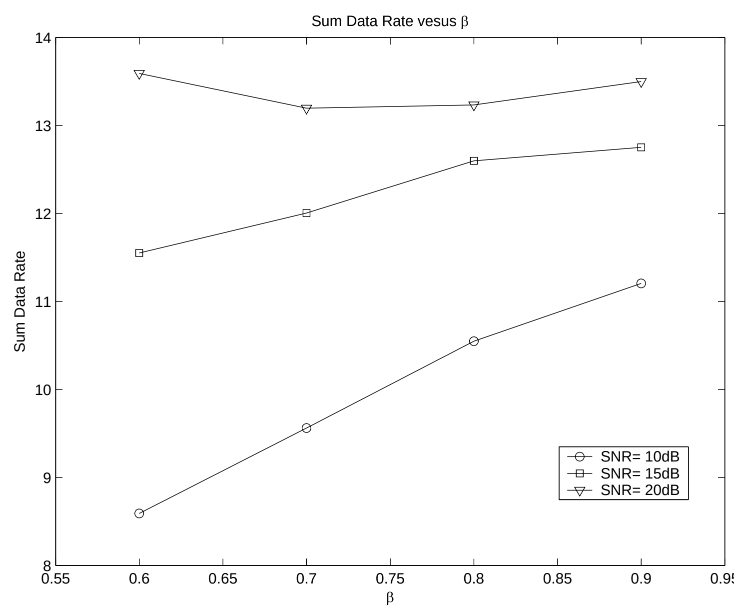

Figure 4.11: Sum data rate versus β in the GGT method

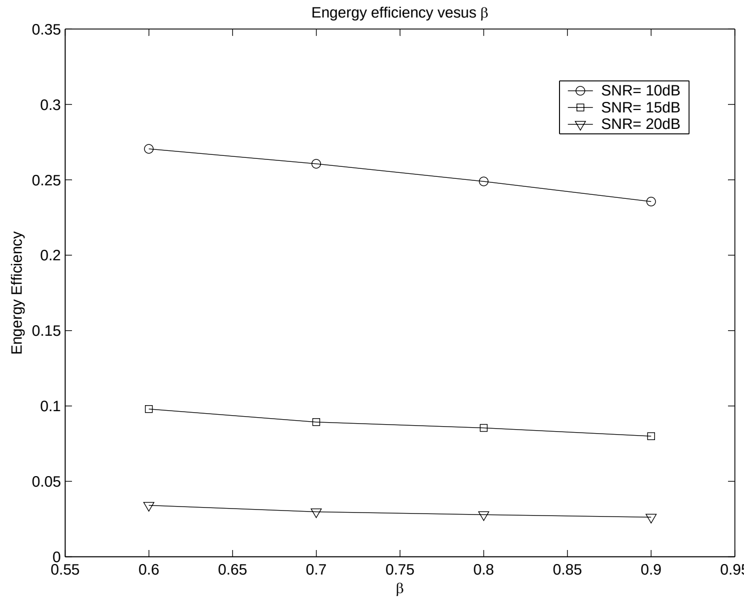

Figure 4.12: Energy efficiency β in the GGT method

4.5 Summary

In this chapter power control for MIMO-OFDM ad hoc networks were discussed. We investigated the problem of subcarrier assignment and power distribution among multiple antennas for point-to-point links in a network without base stations. A subcarrier assignment scheme was proposed which selects a set of subcarriers for each link so that a high data rate can be achieved and co-channel interference can be mitigated. The power management in a MIMO-OFDM ad hoc network was also built into a non-cooperative game in which each link calculates its optimal power allocation vector in order to maximize the net utility. It was also shown that the designed utility function facilitates subcarrier assignment schemes by using a tunable pricing factor, which helps a link to admit or drop subcarriers in a soft and adaptive fashion.

5. Conclusion

In this dissertation, the topic of power management in MIMO-OFDM ad hoc networks was addressed. Existing approaches such as multiuser water-filling and gradient projection assign a fixed transmit power to each link and each transmitter node allocates power among different antennas in order to optimize the link capacity or sum data rate. If bad channel conditions existed in some communicating links, those methods are not energy efficient.

We proposed a new technique for power management and interference reduction based upon a game theoretic approach. Utility functions were designed and power allocation in each link was built into a non-cooperative game. To avoid unnecessary power transmission under poor channel conditions, a mechanism of shutting down inefficient links was integrated into the game theoretic approach. Simulation results showed that under the constraint of a fixed total of transmit power, if the proposed approach were allowed to shut off deficient links, the remaining links would still achieve the highest sum data rate and energy efficiency.

Two kinds of link shut-down mechanism were presented in this dissertation. The first one was called hard shut-down, because once the transmit node decided to shut down, the node will not resume transmission no matter how the interfering channels change. The other mechanism was called soft shut-down, in which the transmit power was related to that link's pricing factor and the interference it was exposed to. With this mechanism, the transmit power can change adaptively in response to the condition of interference.

We also investigated the problem of subcarrier assignment and power distribution among multiple antennas for point-to-point links in a network without base stations. A subcarrier assignment scheme was proposed which selects a set of subcarriers for each link so that a high data rate can be achieved and co-channel interference can be mitigated. The power management in a MIMO-OFDM ad hoc network was also built into a non-cooperative game in which each link calculates its optimal power allocation vector in order to maximize the net utility. It was also shown that the designed utility function facilitates subcarrier assignment schemes by using a tunable pricing factor, which helps a link to admit or drop subcarriers in a soft and adaptive fashion.

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