AC Power Analysis
3-Phase

Prepared for

Electrical Engineering Laboratory II, ECE L302

by

Mohammed Muthalib

Center for Electric Power Engineering

Drexel University

(http://power.ece.drexel.edu)

Outline

- Power Generation
- Fundamental Concepts (recap)
  - 1-Phase: V, I, S,P, Q, PF
- Balanced 3-phase
- Line-line vs. line-neutral voltage
- 3-phase power
  - $S_{3\Phi}$, $P_{3\Phi}$, $Q_{3\Phi}$,
- Why 3-phase?
- This weeks experiment
- Power lab safety

Generation

Nuclear plant

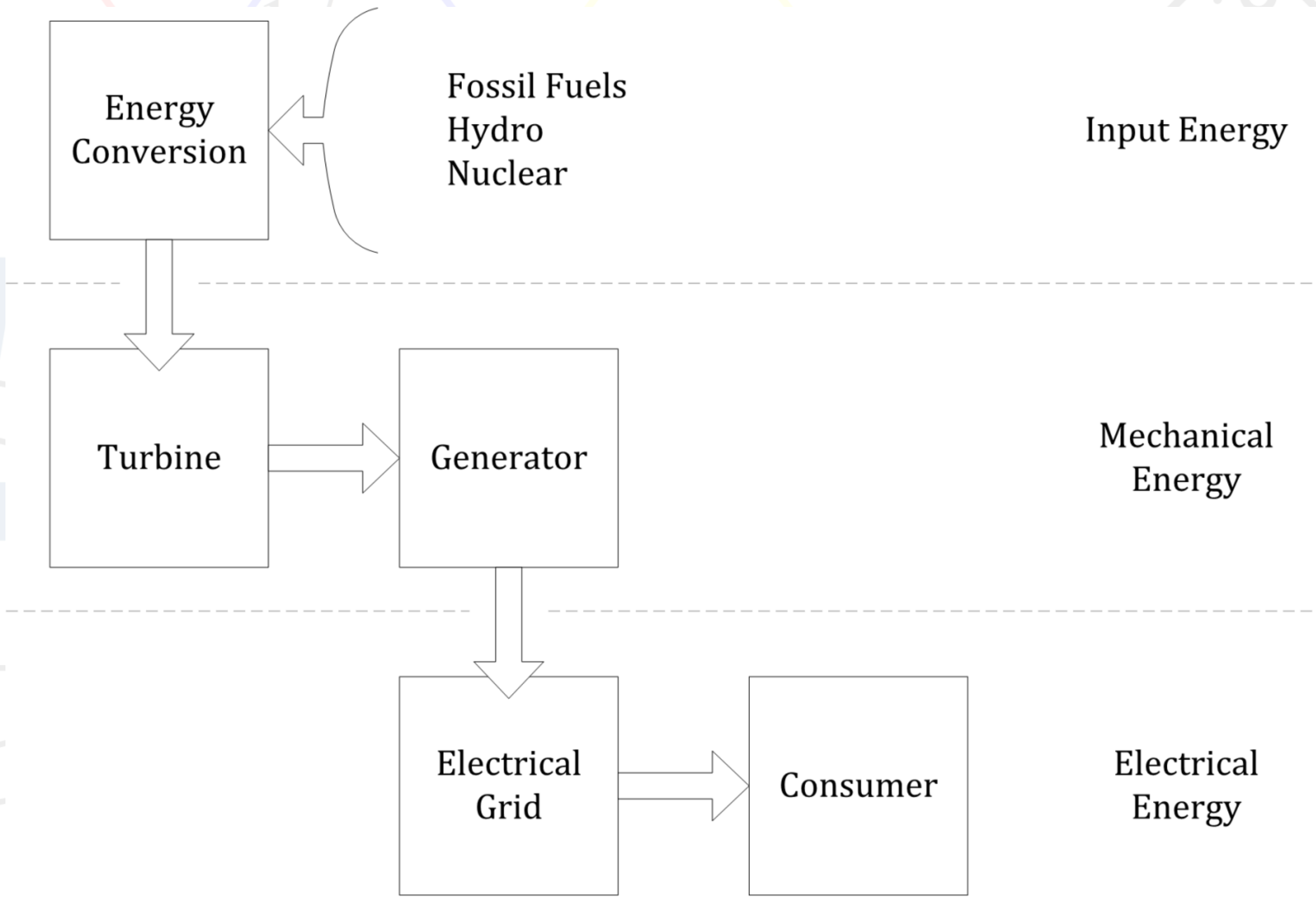
Hydro plant

Coal plant

Fossil fuel plant

Generation

- Conservation of energy



Generation

- Fossil Fuel:
  - Burn fuel → Steam→ Turbine→ Generator
- Nuclear:
  - Nuclear reaction → Steam→ Turbine→ Generator
- Hydro:
  - P.E. of reservoir → K.E. of water→ Turbine→ Generator
- Wind:
  - K.E. of wind → rotational K.E. of blades in turbine→ Generator
- Solar:
  - Solar radiation→ semiconductor energy transfer→ Electron flow
- Solar and wind require storage and AC/DC conversion

Generation

- Turbine – Generator
  - Turbine provides mechanical torque to the rotor of the generator
  - Stator has coils that react to the electro-magnetic field variations

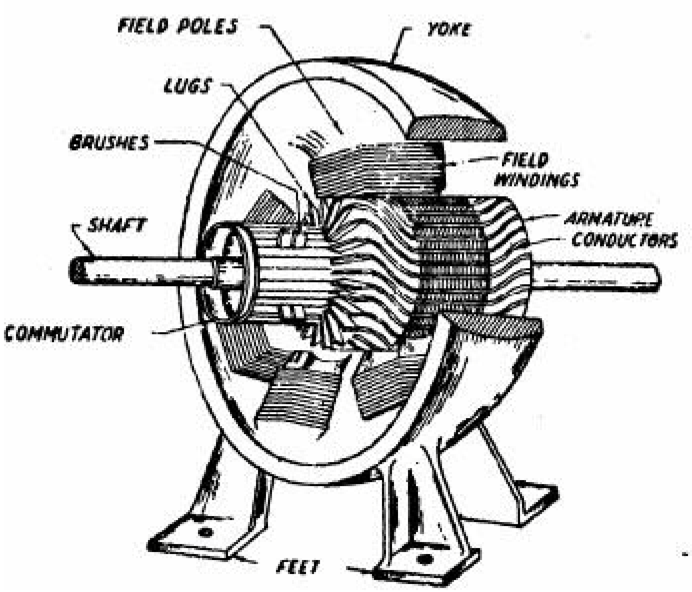


Fig.3.10 Practical DC machine parts .

http://dcacmotors.blogspot.com/2009/03/basic-generator.html

Generation

❑ 3-phase generation

- Stator has 3 coils wound 120° out of phase

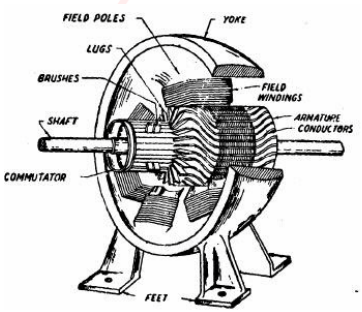
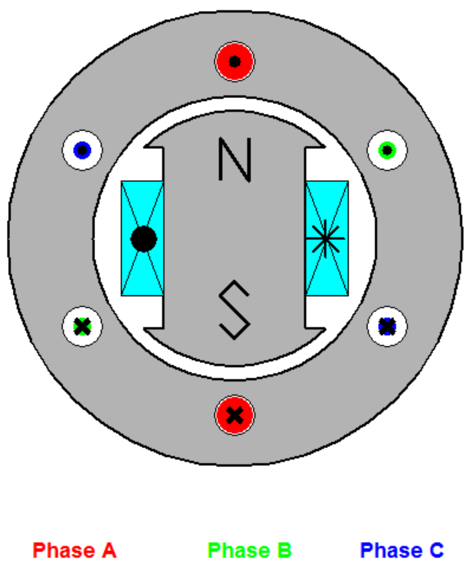
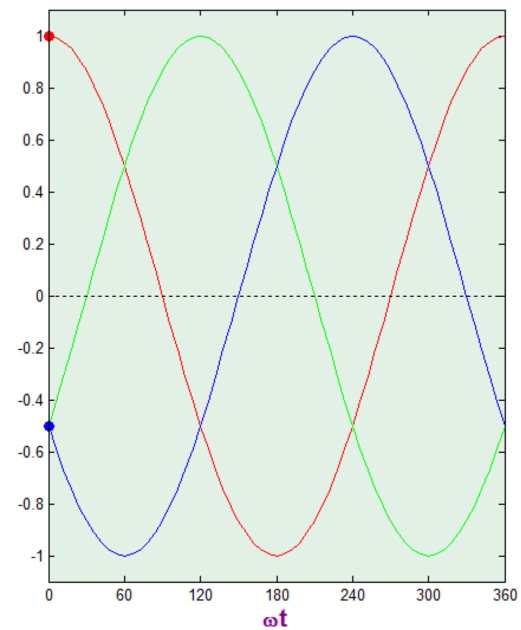


Fig.3.10 Practical DC machine parts .





ECE Department, University of Minnesota
http://www.ece.umn.edu/users/riaz/animations/alternator.html

Fundamental Concepts

AC voltage and current

- Sinusoidal time varying waveforms

$$v(t) = V_{peak}\sin(\omega t + \theta_V)$$

$$i(t) = I_{peak}\sin(\omega t + \theta_I)$$

θ_V –voltage phase angle

θ_I –current phase angle

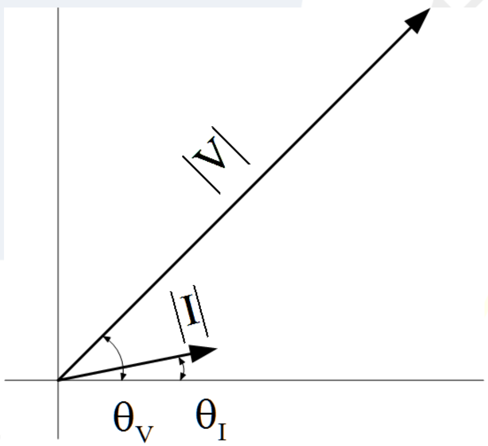
- Phasor representation (time invariant)

$$\mathbf{V} = |V|\angle\theta_V = |V|e^{j\theta_V}$$

$$\mathbf{I} = |I|\angle\theta_I = |I|e^{j\theta_I}$$

Where $|V|$ and $|I|$ are rms values

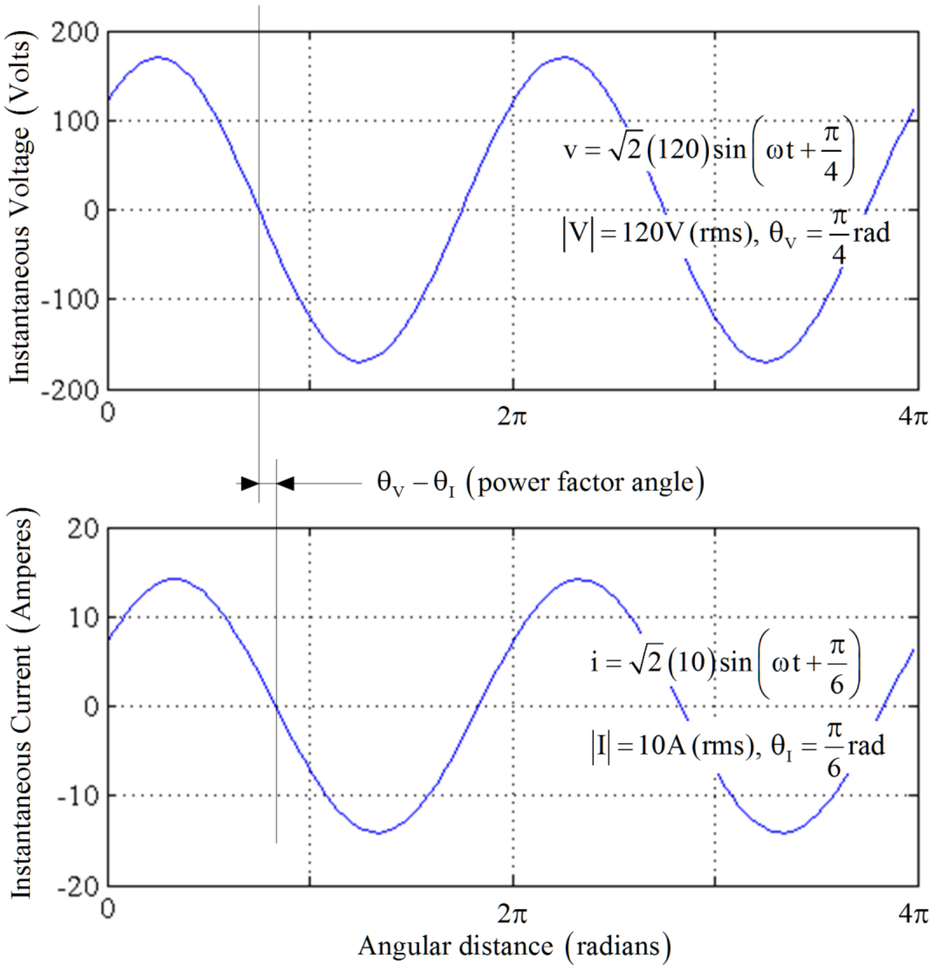
$$V_{peak}\,(\text{amplitude}) = \sqrt{2}\,|V|$$

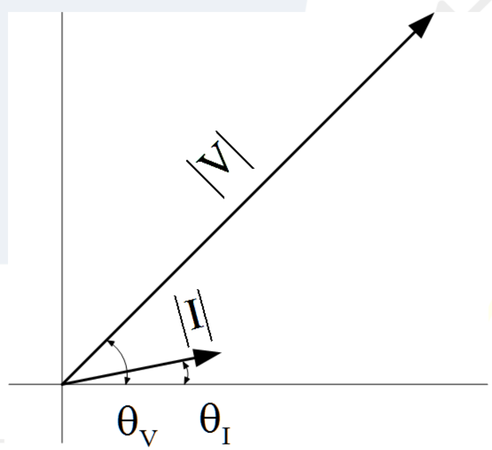


Voltage and current phasors (polar coordinates)

Fundamental Concepts

❑ Voltage and current waveforms





Voltage and current phasors
(polar coordinates)

Fundamental Concepts

- Instantaneous power:
  - Product of v(t) and i(t)

$$v(t) = \sqrt{2}\left|V\right|\sin\left(\omega t + \theta_V\right)$$

$$i(t) = \sqrt{2}\left|I\right|\sin\left(\omega t + \theta_I\right)$$

$$\begin{aligned} p(t) &= v(t) * i(t) \\ &= 2\left|V\right|\left|I\right|\sin\left(\omega t + \theta_V\right)\sin\left(\omega t + \theta_I\right) \end{aligned}$$

use trigonometric identity

$$2\sin u \sin v = \cos(u - v) - \cos(u + v)$$

$$u = \omega t + \theta_V,\;\; v = \omega t + \theta_I$$

$$p(t) = \underbrace{\left|V\right|\left|I\right|\cos\left(\theta_V - \theta_I\right)}_{\text{time invariant (constant)}} - \underbrace{\left|V\right|\left|I\right|\cos\left(2\omega t + \theta_V + \theta_I\right)}_{\text{time varying with frequency } 2\omega}$$

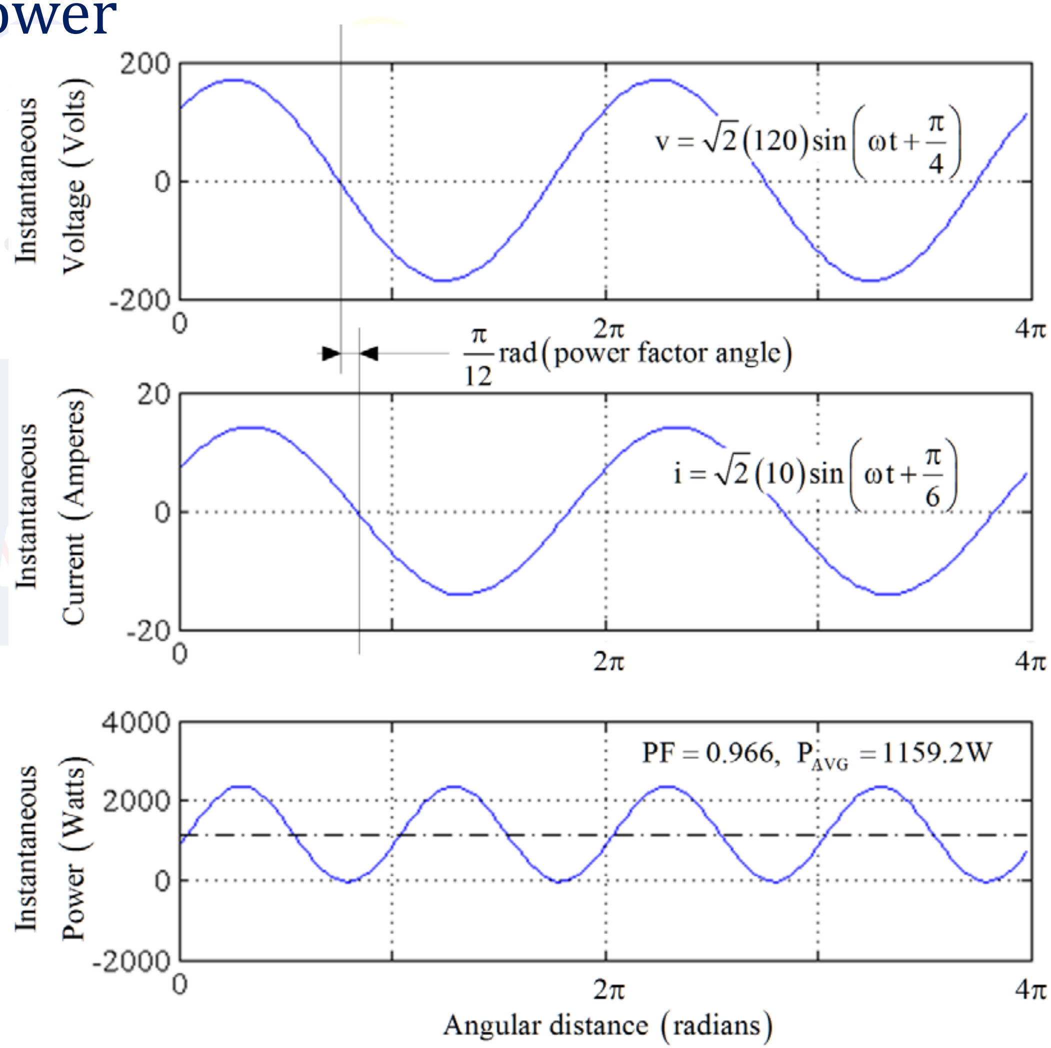
time invariant (constant)

time varying with frequency 2ω

Fundamental Concepts

- Instantaneous power

$$p(t) = v(t) * i(t)$$



Fundamental Concepts

- Complex power:
  - Is a representation of power in a complex vector space
  - Denoted S with units of VA

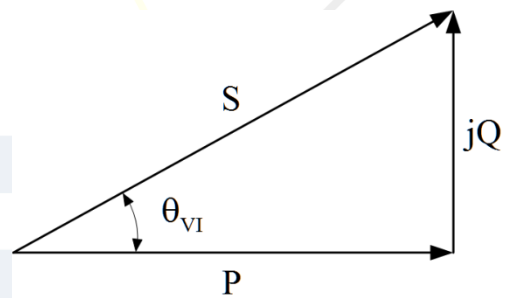
$$S = VI^{*} \; (\text{*denotes complex conjugate})$$

$$S = |V||I|\angle(\theta_V - \theta_I) = |V||I|e^{j(\theta_V - \theta_I)}$$

using Euler's formula

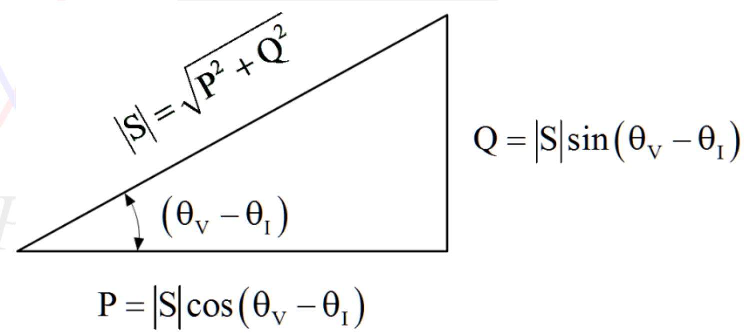
$$S = |V||I|\cos(\theta_V - \theta_I) + j|V||I|\sin(\theta_V - \theta_I)$$

$$= P + jQ$$



- |S| - apparent power

$$|S| = \sqrt{P^2 + Q^2} = |V||I|$$



Fundamental Concepts

- Real power:
  - Real power = Average power

$$P = P_{AVG} = |V||I|\cos(\theta_V - \theta_I)$$

  - Denoted P with units of Watts
  - Power absorbed by the resistive components of the system
- Reactive power:
  - Denoted Q with units of VAR
  - Power absorbed by the reactive components of the system
  - Example, Inductance and Capacitance

$$Q = |V||I|\sin(\theta_V - \theta_I)$$

Fundamental Concepts

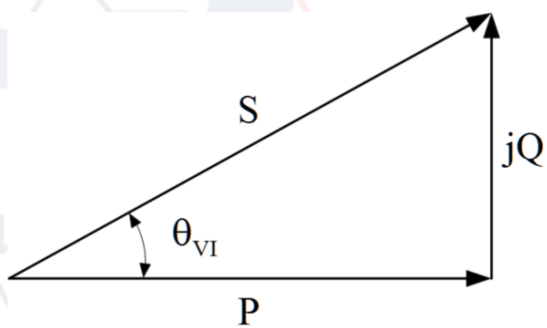
- Power Factor:
  - Is a measure of how effectively a system component draws real power.
  - It is the ratio between real power and apparent power

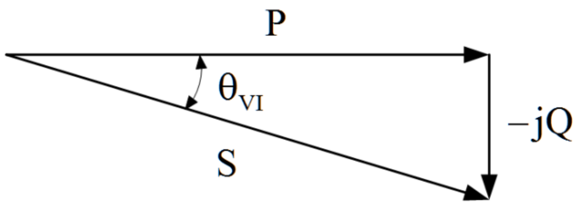
$$PF = \frac{P}{|S|} = \cos(\theta_V - \theta_I)$$

  - PF is presented as a real number between 0 and 1 with a leading/lagging denotation for the PF angle
    - Lagging - current angle lags the voltage angle, $\theta_V > \theta_I$
    - Leading - current angle leads the voltage angle, $\theta_V < \theta_I$

Fundamental Concepts

- Power Factor:
  - Which figure represents a load with a lagging PF?
    - What kind of load has a lagging power factor?
  - Which figure represents a load with a leading PF?
    - What kind of load has a leading power factor?



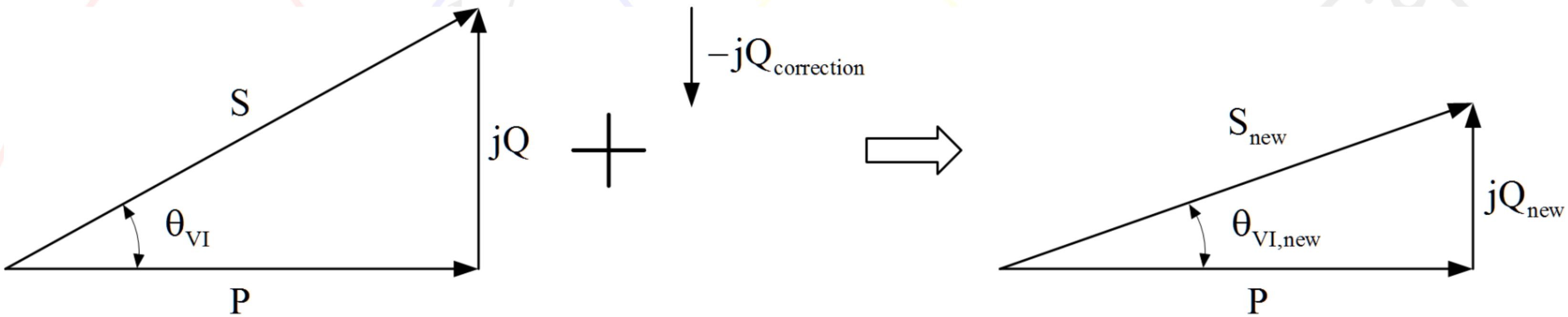


Fundamental Concepts

- PF=1 indicates that all the power consumed in the system is real power.
  - A load with PF=1 emulates a resister
  - Reactive power draw is zero
- Is PFC necessary? Why?

Fundamental Concepts

- PFC by injecting reactive power



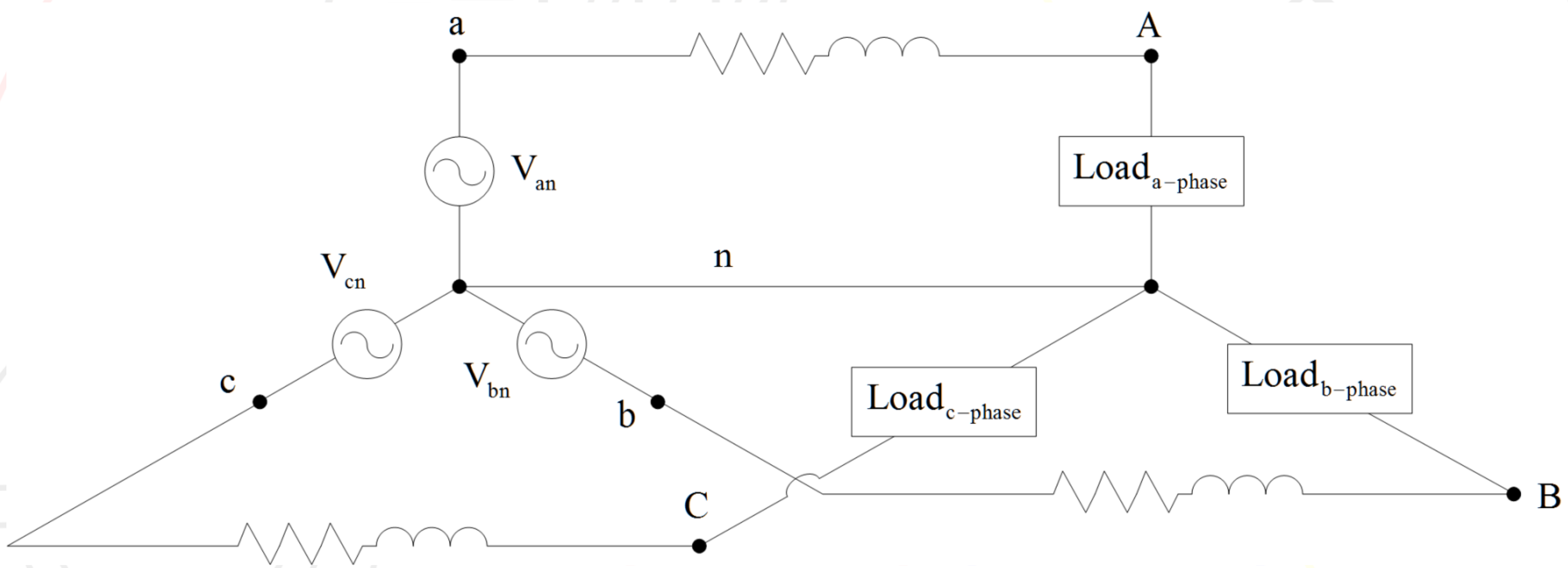
$$S_{new} = P + jQ_{new} = P + j\left(Q - Q_{correction}\right)$$

$$\cos\left(\theta_{VI}\right) < \cos\left(\theta_{VI,new}\right)$$

- Reactive power injection is done by adding capacitive and inductive loads to the system.
  - Capacitive – supply reactive power
  - Inductive – consume reactive power

Balanced 3-phase

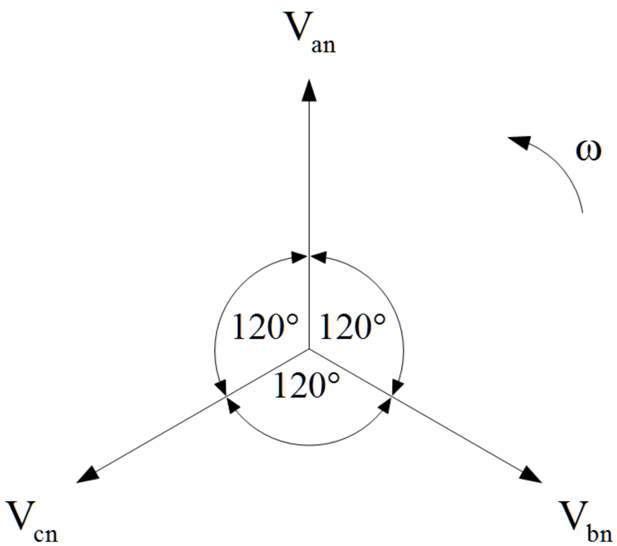
- Balanced 3-phase circuit
  - All components, loads, lines, etc. are assumed to be identical

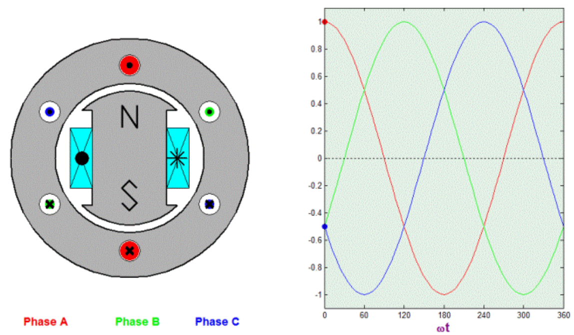


  - Balanced 3Φ circuits are analyzed through a single phase equivalent analysis

Balanced 3-phase

- Balanced 3-phase voltages





$$V_{an} = \sqrt{2}\left|V_{an}\right|\sin\left(\omega t + 0°\right) \quad \text{Volts}$$

$$V_{bn} = \sqrt{2}\left|V_{an}\right|\sin\left(\omega t - 120°\right) \quad \text{Volts}$$

$$V_{cn} = \sqrt{2}\left|V_{an}\right|\sin\left(\omega t + 120°\right) \quad \text{Volts}$$

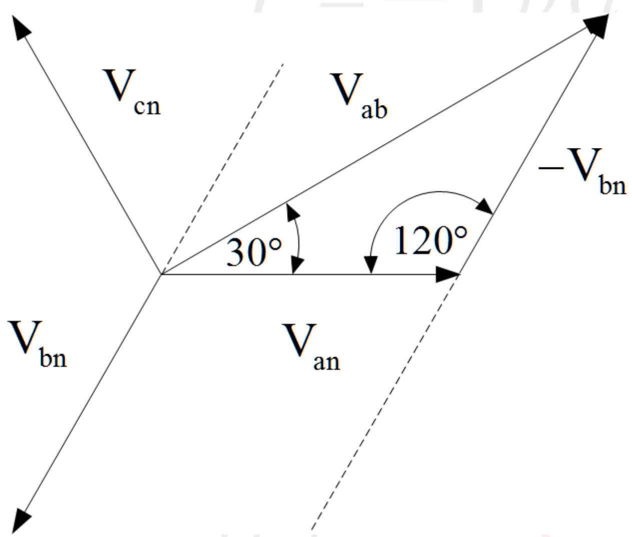
$$I_a = \sqrt{2}\left|I_a\right|\sin\left(\omega t + \theta_I + 0°\right) \quad \text{Amps}$$

$$I_b = \sqrt{2}\left|I_a\right|\sin\left(\omega t + \theta_I - 120°\right) \quad \text{Amps}$$

$$I_c = \sqrt{2}\left|I_a\right|\sin\left(\omega t + \theta_I + 120°\right) \quad \text{Amps}$$

Line-line voltage

- Line-line voltage
  - Voltage measure between phases, V_{ab}, V_{bc}, V_{ca}



$$
\begin{aligned}
V_{ab} &= V_{an} + V_{nb} \\
&= V_{an} + \left(-V_{bn}\right) \\
\left|V_{ab}\right| &= \left|V_{an}\right|\cos\left(30^\circ\right) + \left|V_{bn}\right|\cos\left(30^\circ\right) \\
&= \sqrt{3}\left|V_{an}\right| \\
\angle V_{ab} &= \angle V_{an} + 30^\circ
\end{aligned}
$$

Example:

$$
\begin{aligned}
V_{an} &= 120\angle 0^\circ \quad && V \\
V_{bn} &= 120\angle -120^\circ \quad && V \\
V_{ab} &= 208\angle 30^\circ \quad && V
\end{aligned}
$$

3-phase power

- Real Power
  - $P_{3\Phi} = P_A + P_B + P_C$

$$P_A = |V_A||I_A|\cos(\theta_{V,A} - \theta_{I,A}) \quad \text{Watts}$$

$$P_B = |V_B||I_B|\cos(\theta_{V,B} - \theta_{I,B}) \quad \text{Watts}$$

$$P_C = |V_C||I_C|\cos(\theta_{V,C} - \theta_{I,C}) \quad \text{Watts}$$

$$\theta_{V,B} = \theta_{V,A} - 120°, \quad \theta_{I,B} = \theta_{I,A} - 120°$$

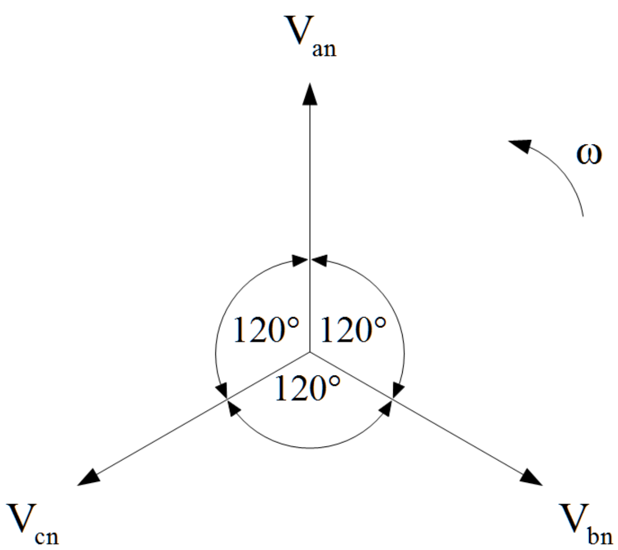
$$\theta_{V,C} = \theta_{V,A} + 120°, \quad \theta_{I,C} = \theta_{I,A} + 120°$$

$$|V_A| = |V_B| = |V_C|, \quad |I_A| = |I_B| = |I_C|$$

$$P_B = |V_A||I_A|\cos((\theta_{V,A} - 120°) - (\theta_{I,A} - 120°))$$

$$= |V_A||I_A|\cos(\theta_{V,A} - \theta_{I,A})$$

$$= P_A$$



Balanced 3-phase voltage phasor

3-phase power

- Real Power

$$P_{3\Phi} = P_A + P_B + P_C \quad \text{Watts}$$

$$P_{3\Phi} = 3|V_{an}||I_a|\cos(\theta_{V,A} - \theta_{I,A}) \quad \text{Watts}$$

$$P_{3\Phi} = \sqrt{3}|V_{LL}||I_L|\cos(\theta_{V,A} - \theta_{I,A}) \quad \text{Watts}$$

- Reactive power

$$Q_{3\Phi} = Q_A + Q_B + Q_C \quad \text{VAr}$$

$$Q_{3\Phi} = 3|V_{an}||I_a|\sin(\theta_{V,A} - \theta_{I,A}) \quad \text{VAr}$$

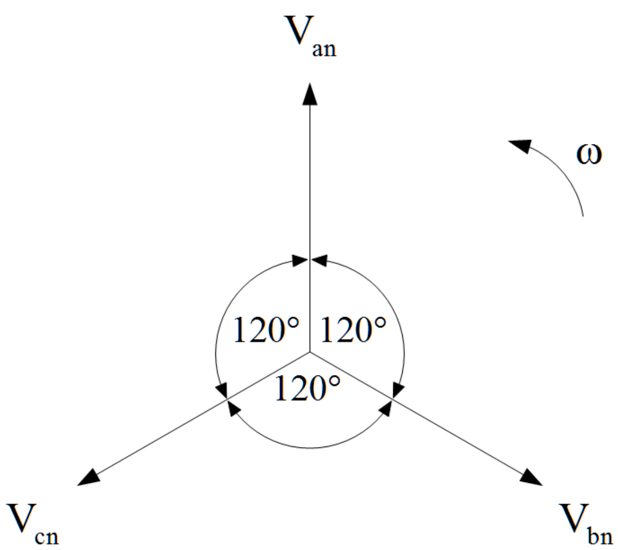
$$Q_{3\Phi} = \sqrt{3}|V_{LL}||I_L|\sin(\theta_{V,A} - \theta_{I,A}) \quad \text{VAr}$$

- Complex Power

$$S_{3\Phi} = 3V_A(I_A)^* \quad \text{VA}$$

$$S_{3\Phi} = P_{3\Phi} + jQ_{3\Phi} \quad \text{VA}$$

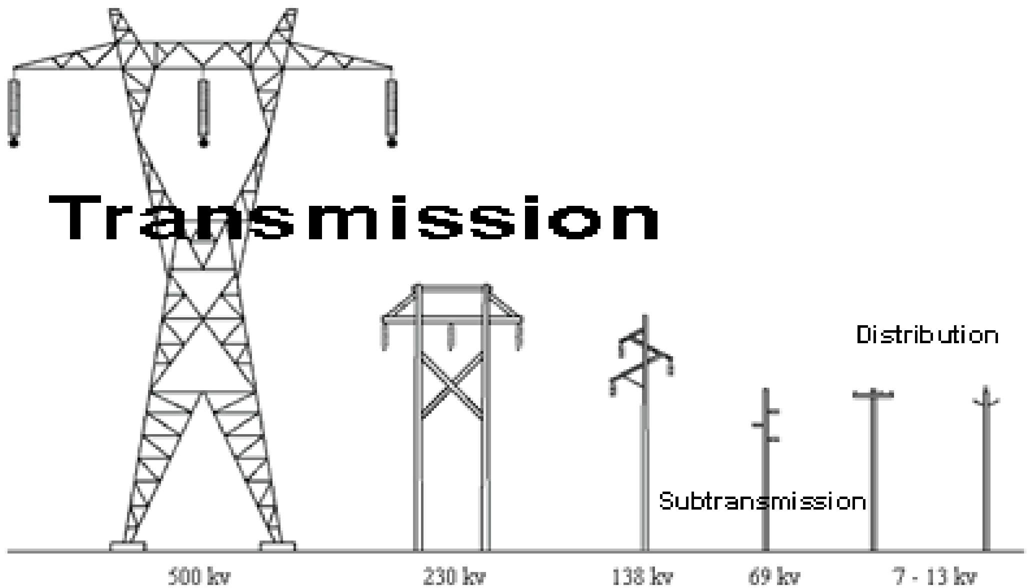
$$|V_{LL}| = |V_{ab}| = \sqrt{3}|V_{an}|$$



Balanced 3-phase voltage phasor

Why 3-phase?

- Electrical energy has to be transmitted/distributed over long distances
  - Cables are heavy (engineering problem to string up in the air) and lossy



https://www.osha.gov/SLTC/etools/electric_power/illustrated_glossary/transmission_lines.html

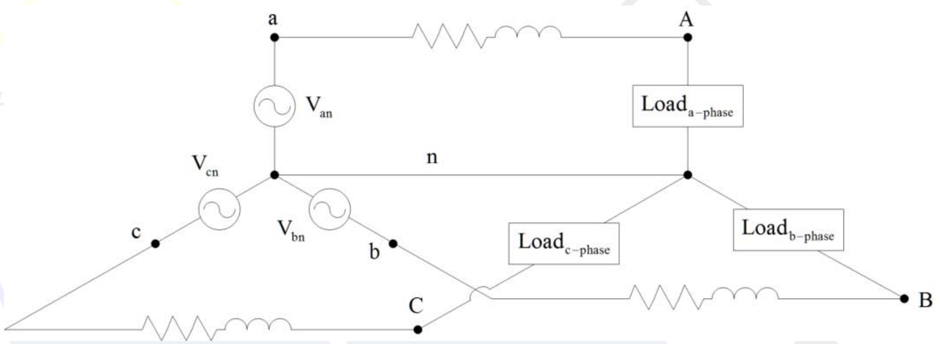
Why 3-phase?

- Neutral Current (balanced 3-Φ)

$$I_a = \sqrt{2}|I_a|\angle(\theta_I + 0^\circ) \quad \text{Amps}$$

$$I_b = \sqrt{2}|I_a|\angle(\theta_I - 120^\circ) \text{Amps}$$

$$I_c = \sqrt{2}|I_a|\angle(\theta_I + 120^\circ) \text{Amps}$$



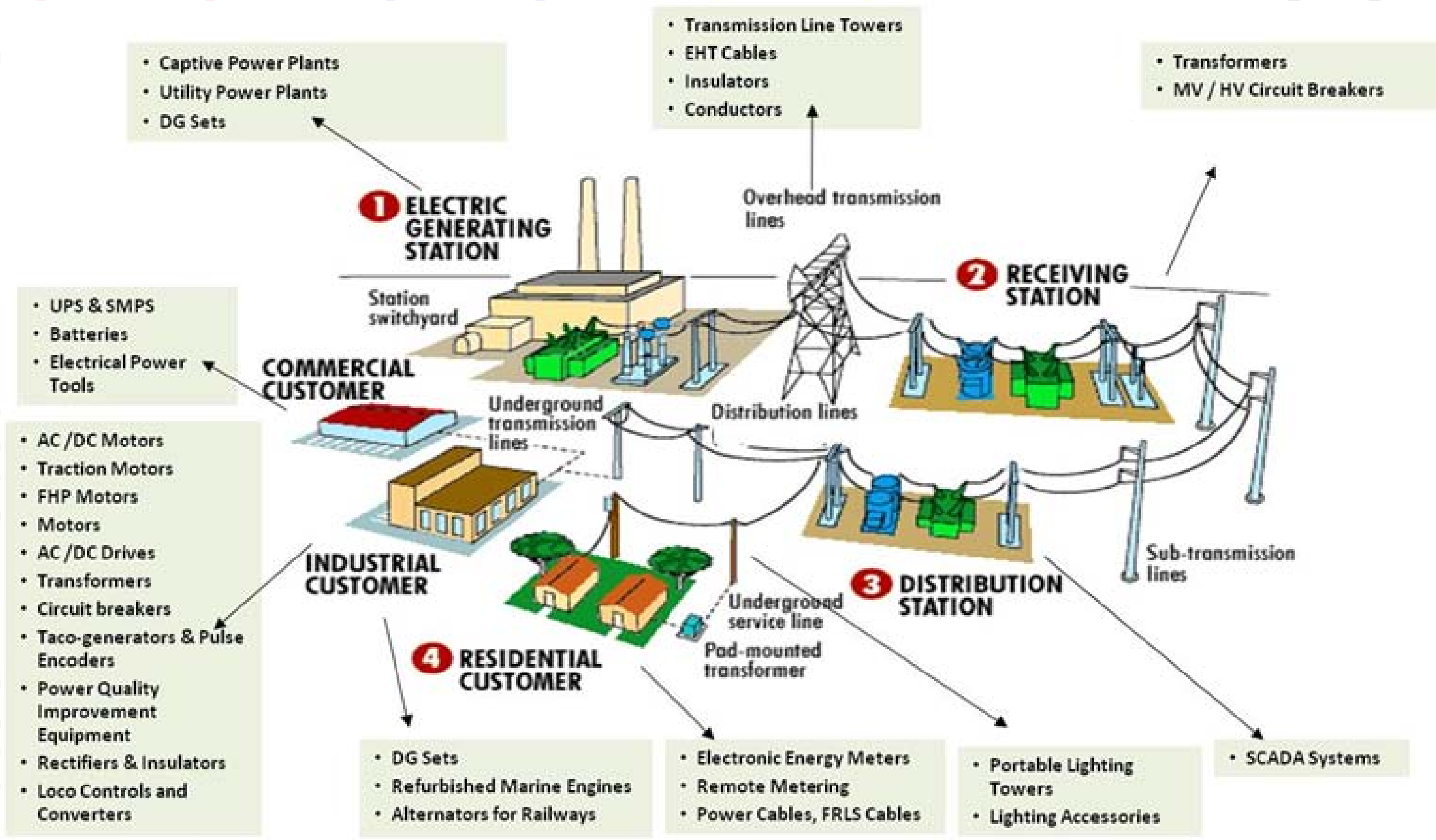
$$\begin{aligned} I_n &= I_a + I_b + I_c \\ &= \sqrt{2}|I_a|\left(\cos(\theta_I) + \cos(\theta_I - 120^\circ) + \cos(\theta_I + 120^\circ)\right) \\ &+ j\sqrt{2}|I_a|\left(\sin(\theta_I) + \sin(\theta_I - 120^\circ) + \sin(\theta_I + 120^\circ)\right) \\ &= \sqrt{2}|I_a|\left(\cos(\theta_I) + 2\cos(\theta_I)\cos(-120^\circ)\right) \\ &+ j\sqrt{2}|I_a|\left(\sin(\theta_I) + 2\sin(\theta_I)\cos(-120^\circ)\right) \\ &= 0 \end{aligned}$$

Why 3-phase?

- Transmission of power in balanced 3-phase can be done without a neutral conductor
- More efficient and economical since:
  - less losses from conductor
  - Less cost with respect to material and construction
- NOTE:
  - Distribution systems are unbalanced and require a neutral conductor
  - Connect back to local electrical substations
  - Supplied through distribution transformers

Electrical Supply

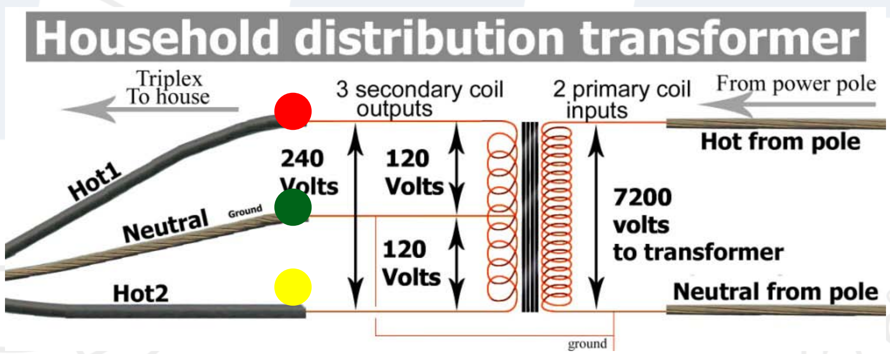
❑ Transmission/Distribution

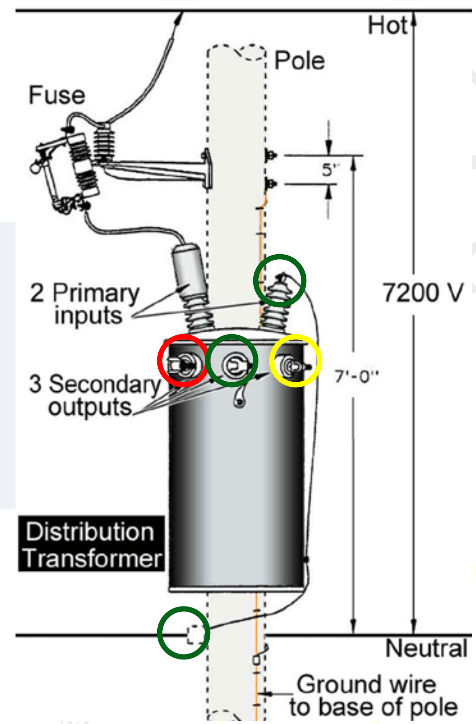


http://www.avalon-energy.com/sample.aspx?C1=28

Electrical Supply

- Residential connection

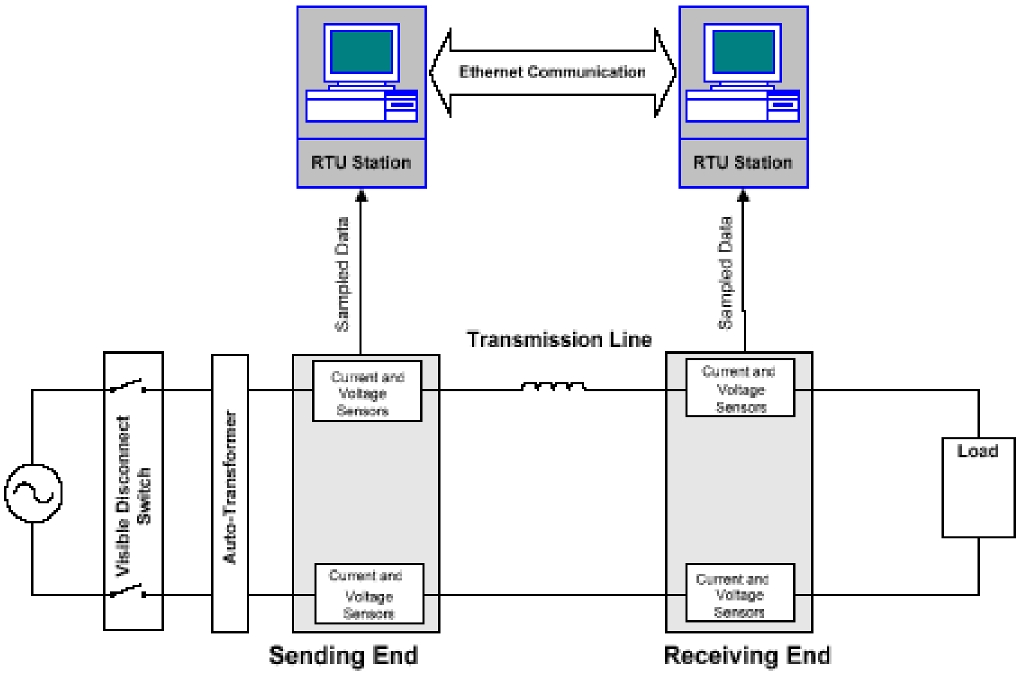




http://waterheatertimer.org/See-inside-main-breaker-box.html

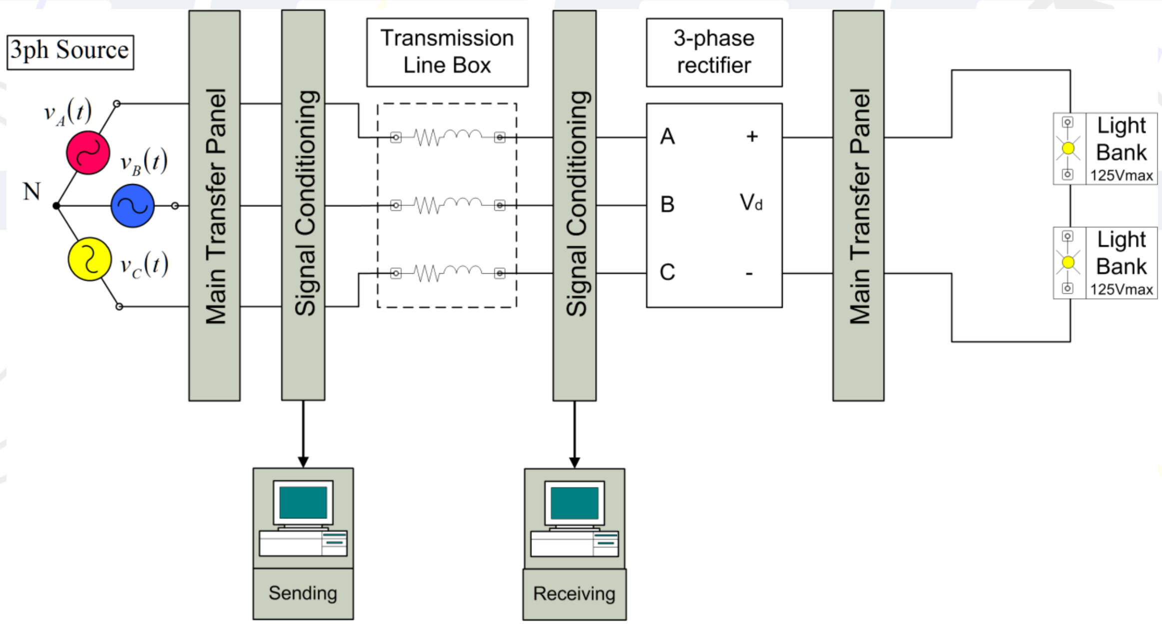
Lab Experiment

- Simulating a 2-bus power system (Transmission)
- Measure voltages and currents; calculate power
- Observe effects of loading



Lab Experiment

- Test system with 3 load types
  - Resistive Load (4 – 20 bulbs)
  - RL (20 bulbs + 0 - 5 inductors)
  - RC (20 bulbs + 0 - 4 capacitors)
  - Nonlinear Load



Lab safety

- You will be dealing with voltages and currents of 120V and 25A
- Adherence to safety and conduct guidelines is imperative
- Please read the Power Lab Safety document
- Please watch lab safety video
  - http://power.ece.drexel.edu/videos/

Thank you

Questions are most welcome